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ADVANCES IN
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AND OPTICAL PHYSICS

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4. Trapping Atoms With Radio Frequency Adiabatic Potentials 181

Hélène Perrin and Barry M. Garraway

1. Introduction 182
2. Basic Concepts 185
3. Introduction to Magnetic Resonance in Classical and Quantized Descriptions 190
4. Adiabatic Potentials for rf-Dressed Atoms 203
5. Examples of Adiabatic Potentials 212
6. Time-Averaged Adiabatic Potentials 226
7. Multiple rf Fields 233
8. Practical Issues With rf-Dressed Atom Traps 244
9. Conclusion 256

Acknowledgments 257
References 257

5. Quantum Control of Optomechanical Systems 263

Sebastian G. Hofer and Klemens Hammerer

Notation 264
1. Introduction 265
2. Cavity-Optomechanical Systems 269
3. Pulsed Entanglement Creation and Verification 290
4. Time-Continuous Quantum Control 301
5. Experimental Implementation of a Kalman Filter 330
6. Conclusion 338

Appendices 338
Appendix A. Quantum Stochastic Calculus 338
Appendix B. Quantum Filtering and Control 346
References 364


Matthias Meister, Stefan Arnold, Daniela Moll, Michael Eckart, Endre Kajari, Maxim A. Efremov, Reinhold Walser, and Wolfgang P. Schleich

1. Introduction 376
2. Efficient Description of the Time Evolution of a BEC 378
3. Application of the Affine Approach to Numerical Simulations 394
6. Halogen-Containing Polyatomic Molecules for Plasma Technologies 594
7. Multidimensional Vibrational Dynamics in the Local Approximation 601
8. Biologically Relevant Molecules 609
9. Clusters 624
10. Conclusions and Outlook 635
Acknowledgments 639
References 639

Index 659
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Multidimensional coherent spectroscopy, first introduced in nuclear magnetic resonance, was later transferred to the optical domain based on the nonlinear response of the explored medium. By relying on the enormous progress in the ultrafast laser technology, it is today a well-established investigation tool, such that many ultrafast spectroscopists use it as their primary technique. It probes the structure and dynamics of the investigated medium by exciting it with a sequence of phase-coherent pulses and recording its response as two or more delays are varied. It excels at determining if the medium energy levels resonant with the exciting radiation are coupled, overcoming the effects of inhomogeneous broadening and disentangling congested resonances by spreading them in two dimensions. The contribution of Li and Cundiff introduces the basis of two-dimensional coherent spectroscopy in order to probe electronic transitions in atomic vapors and semiconductor heterostructures. The atomic vapors provide an ideal model system to test the technique. At the same time, it provides unexpected results that arise from interactions between atoms in the vapor.

The invention of the laser and the development of nonlinear optics have opened the way to the controlled generation of quantum states of light, that is, states endowed with nonclassical features. A very exemplary quantum state is the twin beam, namely, the maximally entangled bipartite state of light generated by the second-order nonlinear process denoted as parametric down conversion. Twin beams exhibit temporal and spatial correlations as well as perfect photon-number correlations. In Chapter 2 Allevi and Bondani review their recent experimental and theoretical explorations, mainly in the high-intensity regime with a large number of correlated photons created on the twin beams. A precise determination of the nonlinear phenomenon and of this dependence on the laser intensity allows exploring how the process features can be optimized and tailored for different applications to quantum technologies.

Chapter 3 by Radulaski, Fischer, and Vuckovic is both a review and a preview of state-of-the-art nonclassical light states such as single photons from quantum dot sources. Quantum electrodynamics in engineered GaAs-based systems is a widely studied field, but the authors’ Stanford experiments are at the cutting edge of the newest developments, including photon blockade effects in optical nanocavities. This chapter includes proposals for the next generation of such emitters.
Ultracold atoms at microkelvin temperatures and below are prepared and exploited within traps, i.e., regions in space having a properly tailored potential. The trap development was an essential step for the initial production of Bose–Einstein condensates. Within the last few years the historical trap geometries have been modified in order to achieve a higher control on the atomic spatial distribution. In this volume Perrin and Garraway present the atomic traps based on radiofrequency adiabatic potentials producing ad-hoc targeted spatial distributions. The whole presentation is very didactic and represents a sort of manual for any further development of those traps. The text includes the essential ingredients of adiabaticity, magnetic resonance, and time-averaged adiabatic potentials, discusses subtle issues as the influence of radiofrequency multiphoton transitions, and also describes practical issues for the construction of radiofrequency adiabatic potentials.

Quantum optomechanics, where optics and mechanical response are combined together, is a research domain, experiencing a fast growing progress within the last few years. The basic idea is to extend to large objects, from nanomirrors to gigantic mirrors, the quantum control reached by the atomic physics community for atoms, ions, and molecules. While initially it relied mainly on the transfer to a different domain of concepts and tools developed by the ultracold atom community, at the present new research directions are developed. The contribution by Hofer and Hammerer brings together the fields of quantum optomechanics and quantum control theory, exploring the entanglement-enhanced quantum control of the explored systems. They derive a protocol leading to optimal optomechanical feedback cooling and also introduce an optomechanical teleportation scheme able to transfer an arbitrary quantum state from a laser pulse onto the mechanical system. The theoretical presentation is fully self-consistent with detailed theoretical derivations and several appendices, recalling some basic results or approaches.

Bose–Einstein condensates (BECs) in time-changing, especially rotating, traps are the topic of the chapter by Meister, Arnold, Moll, Eckart, Kajari, Efremov, Walser, and Schleich. One of the most important applications of BECs today are quantum sensors based on matter-wave interferometry, in particular for sensing of inertia and gravitational effects. For many of these applications, time-dependent and/or rotating traps are necessary. Traditionally, however, the theoretical description of these systems using the Gross-Pitaevskii equation becomes very hard or even impossible without
effective formalisms. The chapter talks about the development, advantages, and uses of such a formalism that is expected to make many descriptions possible and discusses many potential uses.

Chapter 7 covers the development of optical nanofibers, a novel device which shows strong possibilities for their use in quantum optics and quantum information science. Among the novel nature of these nanofibers are the tight optical mode confinement and propagation free of diffraction. In this article Solano, Grover, Hoffman, Ravets, Fatemi, Orozco, and Rolston started with a brief account of the history of development. The electromagnetic modes and light propagation in nanofibers are treated in some details followed by discussions of the interactions of the nanofibers with atoms. Studies of trapped atoms in the vicinity of nanofibers are included. Of great promise are the potential applications of nanofibers to areas such as quantum optics and quantum information.

Studies of projectile coherence in atomic collision processes have been attracting considerable attention in recent years. Chapter 8 is devoted to a survey of current efforts as reviewed by Michael Schulz. The inherent momentum uncertainty associated with the projectiles could be quite large for fast and heavy ion collisions. The dimension of the target which is coherently illuminated would then be small so that the projectiles are often incoherent relative to the target dimension, resulting in a profound influence on the cross sections. After a short exposition of the experimental methods, the author discussed the coherence effects in ion collisions with the hydrogen molecule including the single and double differential studies and fully differential studies. In addition experiments pertaining to coherence effects in the fully differential cross sections for single ionization and transfer-ionization of helium are addressed.

Although the important roles played by the dissociative electron attachment (DEA) in many areas of fundamental or technological importance, especially gas discharges, plasmas, biological systems, and astrophysical environments, have been long recognized, a basic understanding of this process is only slowly evolving. In the final chapter Fabrikant, Eden, Mason, and Fedor survey the basic physics and the progress made in the past 14 years. Aside from the advanced studies of DEA in conventional physical systems, special mention is made of the recent advances in DEA to biological molecules toward understanding radiation damage. The authors offered an extensive discussion on the future outlook and suggestions for new avenues of approach.
Upon completing this volume one of us (C.C.L.) will leave the editorial team. It is a great pleasure for him to have worked with Paul Berman, Ennio Arimondo, and Susanne Yelin. He is particularly indebted to John Boffard for his invaluable assistance over the past 14 years.

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CHAPTER TWO

Nonlinear and Quantum Optical Properties and Applications of Intense Twin-Beams

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Contents

1. Introduction 50
2. Theory 53
   2.1 Twin-Beam Generation 53
   2.2 Correlations 56
   2.3 Multimode Twin Beam 58
   2.4 Schmidt Modes 62
3. Experiments 65
   3.1 Introduction to the Different Regimes and Detectors 65
   3.2 The Laser System 67
   3.3 Photon-Number Statistics 68
   3.4 Spatio-Spectral Properties 73
   3.5 Measurement of Nonclassical Correlations 85
4. Applications 89
   4.1 Conditional States 89
   4.2 Ghost Imaging 93
5. Conclusion 98
Appendix 99
   A.1 Detection Distributions 99
   A.2 Amplification Distributions 100
Acknowledgments 101
References 101

Abstract

In this review we present the main results of some experimental investigations that we performed in the last 10 years on optical twin-beam states. We explore twin-beam statistical properties and spatio-spectral coherence in different intensity regimes, from the mesoscopic to the macroscopic domain, also including pump depletion. We also characterize the quantum nature of the mesoscopic twin-beam states in terms of different nonclassicality criteria, all of them written for detected photons and demonstrate the
The generation of sub-Poissonian non-Gaussian states by conditional operations. Finally, we address a ghost-imaging protocol implemented with twin-beam states in the macroscopic regime.

1. INTRODUCTION

With the invention of the laser in 1960, the generation of new states of light became feasible, together with their application to novel systems, almost covering any topic of physical investigations. Among the new fields opened by laser science, nonlinear optics has a major role, exploiting the properties of coherence, directionality, and high intensity of laser sources. As a matter of facts, although nonlinear optical phenomena were known before the invention of the laser (Lewis et al., 1941), new ones were discovered as a side result of working with high-intensity light pulses (Franken et al., 1961; Maker et al., 1962). Lasers also opened the way to the reproducible realization of optical states, leading to the birth of quantum optics.

When treated with a quantum-mechanical formalism, nonlinear optical phenomena have been found to generate, in some instances, genuine quantum states of light, that is, states endowed with nonclassical features (Rarity et al., 1990). Possibly, the most exemplary quantum state is the twin beam (TWB), namely the bipartite state of light generated by the second-order nonlinear process of parametric down-conversion (PDC) from vacuum. Indeed, TWB is the maximally entangled bipartite state.

The first theoretical investigation of PDC dates back to 1968 (Giallorenzi and Tang, 1968). Since then, many efforts have been devoted to the generation and characterization of TWB states in different intensity regimes. Many experiments were realized at the single-photon level thanks to the existence of avalanche photodetectors (Sciarrino et al., 2011). In the macroscopic domain, TWBs have been obtained from parametric oscillators (Gao et al., 1998; Heidmann et al., 1987; Ribeiro et al., 1997), from seeded parametric amplifiers (Aytür and Kumar, 1990; Smithey et al., 1992) and from traveling-wave optical parametric amplifiers (Bondani et al., 2007; Jedrkiewicz et al., 2004; Vasilyev et al., 2000).

As a consequence of being originated by the PDC process, TWBs are well known to exhibit temporal and spatial correlations as well as perfect photon-number correlations (Mandel and Wolf, 1995).

The first experimental measurements of spatial correlations in TWBs, aimed at determining the size of the coherence areas in the transverse planes,
date back to the 1990s (Jost et al., 1998; Malygin et al., 1985) and early 2000s (Haderka et al., 2005a; Mosset et al., 2004). Up to now measurements were performed in the single-photon regime on TWB containing one photon at most by using scanned single-photon detectors (Malygin et al., 1985; Molina-Terriza et al., 2005; Monken et al., 1998), photomultipliers operating in Geiger mode (Malygin et al., 1985), avalanche photodiodes (Molina-Terriza et al., 2005; Monken et al., 1998), intensified CCD cameras (Haderka et al., 2005a; Jost et al., 1998), or electron multiplying CCD cameras (Edgar et al., 2012). The experimental results can be compared to a well-established theory properly working in this regime (Grayson and Barbosa, 1994; Hamar et al., 2010; Joobeur et al., 1994, 1996).

TWBs can also be generated in a much higher intensity domain (Brambilla et al., 2004), where they show coherence areas that become visible in single-shot images (Bondani et al., 2012; Jedrkiewicz et al., 2004). In the high-gain parametric process, the size of the coherence areas exhibits dependence on the gain (Jedrkiewicz et al., 2004). In particular, at very high intensity, the evolution of the pump field during the PDC process cannot be neglected and pump depletion affects the TWB properties, such as their internal structure, intensity, and correlations (Allevi and Bondani, 2014; Allevi et al., 2014a,b, 2015).

Although the description of the PDC process is the same at the different intensities, so that, for instance, TWB states are endowed with entanglement at any mean value, the measurement of some properties become very critical at high intensities. For instance, entanglement criteria based on direct detection of photon numbers can be more or less effective according to the intensity regimes.

TWB states have a naturally multimode structure that arises from several features: first of all, the possibility of the process to occur out of phase matching conditions, which depend on crystal length; second, the nonmonochromaticity of the pump field, which is unavoidable in the pulsed regime. The description of the multimode generated field can be obtained by expanding the field either in the basis of plane waves, in which all the modes are monochromatic, or in the basis of Schmidt modes, which are polychromatic. The two approaches address different properties of the TWB.

We note that the multimode structure of the TWB can be either a limitation or a resource for applications. For instance, single-mode TWBs would be desirable for applications to quantum information (Grice et al., 2001), while multimode fields are more effective for the generation of
conditional nonclassical states (Allevi et al., 2010; Lamperti et al., 2014). For this reason, it is interesting to exploit the dependence of the number of modes on the interaction parameters to optimize the tailoring of the states.

This chapter reviews some recent results on generation, characterization, and application of TWBs in the high-intensity regime up to pump depletion, experimentally achieved in the laboratory of Quantum Optics at University of Insubria. Such results have been obtained by means of a ps-pulsed laser source regeneratively amplified at 500 Hz, which delivers pulses with high-peak intensities (up to some GW/cm$^2$) at 349 nm (third harmonics). All over the years, this source allowed us to produce bright TWBs at each pump pulse. Indeed, ps-pulsed lasers represent the best sources to observe intense TWBs, as they can produce pulses with high-peak intensities in a limited pulse spectrum (up to 1 nm), thus preventing the occurrence of the dispersion effects typical of femtosecond laser sources. In our research activity, the same laser was also used to investigate the almost unexplored mesoscopic domain. Actually, this regime represents an interesting compromise between the single-photon level, where single-photon pairs are produced, and the macroscopic one, in which bright TWB states are generated. In the mesoscopic domain, where sizeable numbers of photons per pulse are produced, the TWB states are more robust than single-photon pairs with respect to losses, but still preserve, and easily exhibit, the nonclassical properties that make TWBs useful for several applications to quantum technologies. At the moment, the complete investigation of the mesoscopic regime is limited by the performance of the available detectors. Although during the last 15 years different kinds of photon-number resolving detectors have been developed and tested, the ideal solution is still far to come. In this review, we present the characterization of mesoscopic TWBs achieved by means of hybrid photodetectors, which are a particular class of commercial photon-number resolving detectors operating at room temperature. Such a characterization is not only limited to the investigation of the statistical properties, but also extended to the nonclassical features, which represent a useful resource in quantum state engineering for applications to quantum information and metrology. For instance, it is well known that TWBs can be used to produce nonclassical states by means of conditional measurements based on the existence of correlations between signal and idler. In particular, when a given number of photons is detected in the idler, the state of the signal is irreversibly changed from classical to nonclassical and from Gaussian to non-Gaussian. In this review, we show that sub-Poissonian states can be generated by performing conditional measurements on mesoscopic TWBs,
where the multimode nature of TWBs represents a crucial resource. The same property is of paramount importance for the other application, we present in this review, namely the implementation of a ghost-imaging protocol. Indeed, the visibility and the signal-to-noise ratio, which are the fundamental indicators of the quality of the imaging protocols, are strongly affected by the number of both spatial and temporal modes characterizing the TWB states. Finally, we remark that the investigations of TWBs at different intensities regimes gave us a better knowledge of the nonlinear phenomenon and of the evolution of its properties, and also allowed us to understand in which way the TWB features can be optimized and tailored for the different applications to quantum technologies. The chapter is organized as follows: Section 2 introduces the multimode description of TWB states. Section 3 describes some experimental results obtained in different regimes and with different apparatuses. An initial subsection presents the well-characterized laser source, which represents a key issue for the realization of many experiments. Section 4 shows two relevant applications exploiting different properties of TWB states. Section 5 closes the chapter.

2. THEORY

2.1 Twin-Beam Generation

The process of multimode PDC is described by the interaction Hamiltonian

\[ \hat{H} = \sum_{j}^{\mu} \hat{H}_j \]  

(1)

which is the sum of \( \mu \)-independent contributions

\[ \hat{H}_j \propto \hat{a}_{si,j}^{\dagger} \hat{a}_{id,j} \hat{a}_{pu} + h.c. \]  

(2)

each coupling three monochromatic field modes: pump (\( pu \)), signal (\( si \)), and idler (\( id \)), identified by their frequencies and wavevectors. \( \hat{a}_{l,j}^{\dagger} \) are the field-mode operators satisfying the commutation rules \( [\hat{a}_{l,j}, \hat{a}_{l,i}^{\dagger}] = \delta_{j,i} \) for \( l = si, id \).

In order to obtain the parametric interaction, energy and momentum conservation must be fulfilled

\[ \omega_{pu} = \omega_{si,j} + \omega_{id,j} \]  

(3)

\[ k_{pu}(\omega_{pu}) = k_{si,j}(\omega_{si,j}) + k_{id,j}(\omega_{id,j}). \]  

(4)
The unitary evolution operator in the interaction picture reads

\[
\hat{U} = \exp \left[ -i \tau \left( \hat{a}_{si,j}^\dagger \hat{a}_{id,j}^\dagger \hat{a}_{pu} + \hat{a}_{si,j} \hat{a}_{id,j} \hat{a}_{pu}^\dagger \right) \right],
\]

where \( \tau \) is a rescaled interaction time.

The usual description of PDC in the low-intensity regime is performed within the so-called parametric approximation: considering that the generated TWBs have negligible intensity with respect to the pump, the pump field is assumed as nonevolving during the interaction. Mathematically speaking, the parametric approximation consists in replacing the pump-mode operator \( \hat{a}_{pu} \) with the complex amplitude \( A_{pu} \) of the corresponding coherent state. In phase-matching conditions, we get the evolution operator:

\[
\hat{U} = \exp \left[ \kappa(k_{pu}, k_{si,j}) \hat{a}_{si,j}^\dagger \hat{a}_{id,j}^\dagger - \kappa^*(k_{pu}, k_{si,j}) \hat{a}_{si,j} \hat{a}_{id,j} \right],
\]

where the coupling function \( \kappa(k_{pu}, k_{si,j}) \) depends on the interaction parameters describing the phase-matching geometry in the specific material and on pump intensity.

The state generated from the vacuum by the evolution in Eq. (6) is the single-mode TWB state

\[
|\psi\rangle_j = \sqrt{1 - |\lambda(k_{pu}, k_{si,j})|^2} \sum_n \lambda(k_{pu}, k_{si,j})^n |n\rangle |n\rangle,
\]

\(|n\rangle\) being the state containing \( n \) photons, \( \phi_0 \) being the phase of the pump field, and \( \lambda(k_{pu}, k_{si,j}) \equiv \tanh[\kappa(k_{pu}, k_{si,j})] \exp(i\phi_0) \). The mean photon-number value in the generated signal and idler is \( n_{PDC,j} = \sinh^2[\kappa(k_{pu}, k_{si,j})] \) and the photon-number statistics is a thermal distribution

\[
p_{n,j} = \frac{n_{PDC,j}^n}{(1 + n_{PDC,j})^{n+1}},
\]

so that the TWB state can be rewritten as

\[
|\psi\rangle_j = \sum_n \sqrt{p_{n,j}} \exp(i\phi_0) |n\rangle |n\rangle.
\]

We note that the single-mode TWB displays perfect photon-number correlations and that the wavefunction is written as a nonfactorable superposition of number states, meaning that the TWB state is entangled.
In the case of multimode PDC, we assume that all the contributions in the Hamiltonian in Eq. (1) are equal, so that the generated state is a tensor product of $\mu$ identical twin-beam states (Allevi et al., 2010), differing from each other for the wavevector. The multimode state becomes

$$\lvert \psi_\mu \rangle = \bigotimes_{j=1}^{\mu} \lvert \psi \rangle_j,$$

(10)

where $\lvert \psi \rangle_j$ has the form in Eq. (7) and the mean photon number of all $j$ contributions is a constant, i.e., $n_{\text{PDC},j} \equiv n_{\text{PDC}}$. The mean total number of photons in the multimode state is $N = \mu n_{\text{PDC}}$ and the statistics of photons in each arm of the TWB becomes the multimode thermal distribution

$$p^\mu_n = \frac{(n + \mu - 1)!}{n!(\mu - 1)!(n_{\text{PDC}} + 1)^\mu (1/n_{\text{PDC}} + 1)^n},$$

(11)

We can thus write the multimode TWB in the following compact form

$$\lvert \psi_\mu \rangle = \sum_{n=0}^{\infty} \sqrt{p^\mu_n} \lvert n^\otimes \rangle \lvert n^\otimes \rangle,$$

(12)

where $\lvert n^\otimes \rangle = \delta(n - \sum_{h=1}^{\mu} n_h) \bigotimes_{k=1}^{\mu} \lvert n \rangle_k$ represents the overall $n$ photons coming from the $\mu$ modes that impinge on the detector and $p^\mu_n$ is the multimode thermal distribution in Eq. (11). Note that also for the multimode TWB, perfect correlations in the number of photons are expected.

Eqs. (8) and (11) describe the statistical distributions for the number of photons in the states as they would be detected by a perfect detector. This is obviously not the case for real photodetectors, which reveal light with a nonunit detection efficiency. In Appendix we remind some basics of the description of detectors. Here we only note that the functional form of the statistics of detected photons could in principle be different from that of the photons. Actually, it can be demonstrated (Bondani et al., 2009b; Casini and Martinelli, 2013) that all the classical statistics, obtainable as superpositions of coherent states, are invariant under detection when the positive-operator valued measure (POVM) is Bernoullian (see Appendix). If instead the statistics is nonclassical, such as in the case of Fock or sub-Poissonian states, it will change upon detection.


2.2 Correlations

By looking at the expression for the single-mode TWB, Eq. (9), the pairwise photon-number correlation clearly emerges: the number of photons in the idler is always equal to that in the signal. Actually, in the presence of nonunit detection efficiency (see Appendix) the number of detected photons in the two parties of the TWB may differ, but retain anyway a strong correlation, which is always larger than that of multimode classically correlated states.

The correlation functions $g^{jk}_{n}$ are usually defined in terms of the normally ordered creation and annihilation operators (Glauber, 1965):

$$g^{jk}_{n} = \frac{\langle \hat{n}_{si}^{j} \hat{n}_{id}^{k} \rangle}{\langle \hat{n}_{si} \rangle \langle \hat{n}_{id} \rangle} = \frac{\langle \hat{a}_{si}^{\dagger j} \hat{a}_{id}^{\dagger k} \hat{a}_{si} \hat{a}_{id} \rangle}{\langle \hat{a}_{si}^{\dagger} \hat{a}_{si} \rangle \langle \hat{a}_{id}^{\dagger} \hat{a}_{id} \rangle},$$

where $\hat{a}_{k}$ is the field operator with $k = si, id$ and $\hat{n}_{k} = \hat{a}_{k}^{\dagger} \hat{a}_{k}$ is the number operator. The normally ordered correlation functions $g^{jk}_{a}$ have a well-recognized meaning in connection with coherence properties of light and $n$–photon absorption process (Klyshko et al., 2011; Perina, 1991). However, in a realistic direct-detection scheme, we have only access to the shot-by-shot detected photons and this requires a suitable description of the actual operation performed by the detector. If we assimilate the real detection to a Bernoullian process having efficiency $\eta$, we can express all the operatorial moments of the detected-photon distribution as a function of those of the photon distribution, $\hat{m}_{k}^{n} = \sum_{h=1}^{p} g_{h}(\eta) \hat{n}_{k}^{h}$, where the coefficients $g_{h}(\eta)$ are given in Agliati et al. (2005). In this way we can build correlation functions in analogy with normally ordered ones:

$$g^{jk}_{m} = \frac{\langle \hat{m}_{si}^{j} \hat{m}_{id}^{k} \rangle}{\langle \hat{m}_{si} \rangle \langle \hat{m}_{id} \rangle},$$

where $\hat{m}_{k}$ is the operator describing the actual number of detected photons in the $k$–th arm of the bipartite state. Indeed, the link between the detected–photon correlations and the normally ordered ones is given by

$$g^{jk}_{m} = \sum_{s,t} e_{s,t} g^{st}_{n},$$

where $g_{n}^{00} \equiv 1$ and $e_{s,t}$ depend on the physical parameters of the system under investigation.

The expected results for correlations should be evaluated by taking into account all the realistic experimental conditions, that is, not only the imperfect detection and the imbalance of the arms of the bipartite state, but also its
multimode nature. Thus, we need to derive a suitable theoretical description of the detection process involving multimode states, for which all the \( \mu \) modes in the field are measured shot-by-shot.

By evaluating the expressions in Eqs. (14) for the multimode state in Eq. (12) we obtain a final result that depends only on the number of modes \( \mu \), the mean value of the number of photons \( \langle \hat{n} \rangle \), and the overall detection efficiencies of the two detection chains, \( \eta_1 \) and \( \eta_2 \). In the case of \( \eta_1 = \eta_2 \) and considering correlation functions up to the fourth order, Allevi et al. (2012) reported

\[
\begin{align*}
\mathcal{S}^{11}_{\hat{m}} &= G^1_{\mu} + \frac{\eta}{\langle \hat{m} \rangle}, \\
\mathcal{S}^{21}_{\hat{m}} &= \mathcal{S}^{12}_{\hat{m}} = G^2_{\mu} + \frac{\eta}{\langle \hat{m} \rangle} + \frac{1}{\langle \hat{m} \rangle}^2, \\
\mathcal{S}^{22}_{\hat{m}} &= G^3_{\mu} + 2 \frac{\eta}{\langle \hat{m} \rangle} + \frac{1}{\langle \hat{m} \rangle} + \frac{4 \eta + 2 \eta^2}{\langle \hat{m} \rangle}^2 + \frac{\eta}{\langle \hat{m} \rangle}^3, \\
\mathcal{S}^{31}_{\hat{m}} &= \mathcal{S}^{13}_{\hat{m}} = G^3_{\mu} + 3 \frac{\eta}{\langle \hat{m} \rangle} + \frac{1 + 6 \eta}{\langle \hat{m} \rangle}^2 + \frac{\eta}{\langle \hat{m} \rangle}^3, \\
\end{align*}
\]

where \( \langle \hat{m} \rangle \) is the average number of detected photons and \( G^k_{\mu} = \prod_{j=1}^{k} (j + \mu) / \mu \).

High-order correlations can be used to infer the very nature of bipartite multimode states. A more direct way to establish nonclassicality is to address a suitable quantity satisfying boundary conditions (Klyshko, 1996; Short and Mandel, 1983; Vogel and Welsch, 2006). There are many inequalities that can be used as sufficient criteria for entanglement. Here we consider three criteria, all of them rewritten in terms of measurable quantities (detected photons):

**1** Schwarz inequality \( S > 1 \), where

\[
S = \frac{\langle m_i m_{id} \rangle}{\sqrt{\langle m_i^2 \rangle - \langle m_i \rangle (\langle m_{id}^2 \rangle - \langle m_{id} \rangle)}}.
\]

\[
S = 1 + \frac{\mu}{\mu + 1} \frac{\sqrt{\eta_i \eta_{id}}}{\langle m_i \rangle \langle m_{id} \rangle}.
\]

**2** Noise reduction factor \( R < 1 \), where

\[
R = \frac{\langle (m_i - m_{id})^2 \rangle - \langle m_i - m_{id} \rangle^2}{\langle m_i + m_{id} \rangle}.
\]

\[
R = 1 - 2 \sqrt{\eta_i \eta_{id}} \frac{\sqrt{\langle m_i \rangle \langle m_{id} \rangle}}{\langle m_i \rangle + \langle m_{id} \rangle} + \frac{1}{\mu} \frac{(\langle m_i \rangle - \langle m_{id} \rangle)^2}{\langle m_i \rangle + \langle m_{id} \rangle}.
\]
High-order inequality $B > 1$, introduced by Bondani in Allevi et al. (2012), where

$$B = \langle m_{si} \rangle \langle m_{id} \rangle \frac{g_{m}^{22} - [g_{m}^{13}]_s}{g_{m}^{11}} + \sqrt{\langle m_{si} \rangle \langle m_{id} \rangle} \frac{[g_{m}^{12}]_s}{g_{m}^{11}}, \quad (18)$$

in which we have used the symmetrized quantities $[g_{m}^{hk}]_s = (g_{m}^{hk} + g_{m}^{kh})/2$.

We observe that in terms of detected quantities all the above criteria depend on the intensity of the fields, but in different ways. In fact, $S \to 1$ at increasing number $N$ of photons in the TWB, regardless the values of quantum efficiencies (we remind that $\langle m_{si, id} \rangle = \eta_{si, id} N$). The Schwarz inequality is thus verified at any intensity, but it becomes more and more difficult to experimentally test at increasing intensities. As to $R$, this is the most widely used parameter to test TWB nonclassicality, and it is well known that the inequality is hard to violate at high intensity. The reason is that, while $R < 1$ for any value of $N$ if $\eta_{si} = \eta_{id}$, $R > 1$ for $N < 2 \mu \eta_{si} \eta_{id} / (\eta_{si} - \eta_{id})^2$ if $\eta_{si} \neq \eta_{id}$, which means that there is a threshold in the detectability of entanglement in TWB that depends on the number of modes (the largest, the best) and on the perfect balancing of the efficiencies (the most similar, the best). At variance with $S$ and $R$, $B$ is not sensitive to unbalancing and increases at high intensities.

### 2.3 Multimode Twin Beam

In general, the generated TWBs are intrinsically multimode. However, in the existing literature the TWB states are usually treated as single-mode states, as the calculations are simpler and the nonclassicality criteria useful to characterize such states can be easily defined. In particular, for some physical quantities, such as the quantum discord and the non-Gaussianity, not only the theoretical extension to the multimode case is hardly achievable, but also the experimental measurements are not always easily accessible. Nevertheless, in Section 4 we will show that the multimode nature of TWBs is an important resource for the implementation of some protocols, such as for the generation of sub-Poissonian states. That is why in the following we will consider two different approaches to the description of the multimode nature of TWBs. In the first scenario, the pump beam is treated as a plane-wave undepleted field, whereas in the second one it is allowed to evolve together with signal and idler.


2.3.1 Effect of Phase Mismatch

In order to explore the effects of phase matching on the generation of multimode TWB, we consider the realization of the PDC nonlinear process in a uniaxial crystal, in which the pump field propagates along the normal to the crystal entrance face (Bondani et al., 2012). In Fig. 1, a generic phase-matching condition in space is depicted, along with the definitions of angles that identify the propagation directions inside the crystal. The phase-matching conditions can be written as:

\[
\begin{align*}
    k_{si} \sin \beta_{si} + k_{id} \sin \beta_{id} &= 0 \quad (19) \\
    k_{si} \cos \beta_{si} \sin \theta_{si} + k_{id} \cos \beta_{id} \sin \theta_{id} &= 0 \quad (20) \\
    k_{si} \cos \beta_{si} \cos \theta_{si} + k_{id} \cos \beta_{id} \cos \theta_{id} &= k_{pu} \quad (21)
\end{align*}
\]

The frequency- and phase-matching conditions allow the generation of TWBs on a continuum of frequencies and propagation directions linked to each other by nontrivial relations (Jedrkiewicz et al., 2004). Moreover, the possibility to generate light in conditions of phase-mismatch produces an output field characterized by the simultaneous presence of many modes, both spatial and temporal (Brida et al., 2009b,c). In the far field, the angular and spectral dependence of the output field can be factorized and the multimode character of the states can be considered in time and space separately. In particular, we observe that single-shot images of the output

\[\text{Fig. 1 Scheme of the phase-matched interaction: (x,y)-plane coincides with the crystal entrance face; } \alpha, \text{ tuning angle; } \beta_j, \text{ angles to (y,z)-plane; } \theta_j, \text{ angles on the (y,z)-plane; OA, optical axis. Adapted from Bondani, M., Allevi, A., Andreoni, A., 2012. Ghost imaging by intense multimode twin beam. Eur. Phys. J. Spec. Top. 203 (1), 151–161. ISSN 1951-6401. http://dx.doi.org/10.1140/epjst/e2012-01540-4.}\]
display a “speckle” pattern that can be interpreted as a particular realization of the nonlinear process: the center of each speckle individuates the direction of one of the phase-matched wavevectors, whereas the size of the speckle depends on the angular bandwidth allowed by phase mismatch. Similarly, we can also consider the spectral bandwidth allowed by the frequency mismatch and introduce the concept of temporal modes, which, in the case of a pulsed field, can be viewed as the ratio of the spectral bandwidth of the process to the spectral bandwidth of the pump field (Paleari et al., 2004). The presence of temporal modes reflects on the photon-number statistics of the light inside a single coherence area.

In Fig. 2 we plot a typical single-shot far-field pattern generated in achievable experimental conditions (Bondani et al., 2012). To characterize the multimodal nature of the PDC output, in Mikhailov et al. (2003) we have studied the dependence of the gain of the parametric process, i.e., of the number of photons, of a single-mode TWB as a function of the interaction parameters by analogy with the classical gain of the nonlinear parametric process. We can write the gain for a given choice of mode index $j$ as:

$$\Gamma = \frac{4\gamma^2 |A_{pu}|^2}{(\Delta k \cdot \hat{k}_{si})^2 \left[ \frac{4\gamma^2 |A_{pu}|^2}{(\Delta k \cdot \hat{k}_{si}) (\Delta k \cdot \hat{k}_{id})} - 1 \right]^{-1} \times \sinh^2 \left( \frac{4\gamma^2 |A_{pu}|^2}{(\Delta k \cdot \hat{k}_{si}) (\Delta k \cdot \hat{k}_{id})} - 1 \Delta k \cdot r \right),$$

(22)

where “^” here indicates the versors of the different vectors and \( \gamma \) is the coupling coefficient of the nonlinear process. Mikhailov et al. (2003) have demonstrated that the direction of the phase mismatch is aligned to the bisector between signal and probe so that

\[
\Delta \mathbf{k} \cdot \hat{\mathbf{k}}_{si} = \Delta \mathbf{k} \cdot \hat{\mathbf{k}}_{id} = |\Delta \mathbf{k}| \cos (\psi_{si,id}/2),
\]

\( \psi_{si,id} \) being the angle between signal and probe. We assume

\[
4|A_{pu}|^2 / \left[ (\Delta \mathbf{k} \cdot \hat{\mathbf{k}}_{si})(\Delta \mathbf{k} \cdot \hat{\mathbf{k}}_{id}) \right] \gg 1
\]

and expand the expression in Eq. (22), obtaining:

\[
\Gamma \approx \frac{1}{4} \exp \left( 2\gamma |A_{pu}| r \right) \exp \left( -\frac{|\Delta \mathbf{k}|^2 \cos^2 (\psi_{si,id}/2)}{4|A_{pu}|} r \right). \tag{23}
\]

Note that the value of the gain in phase matching (\( \Delta \mathbf{k} = 0 \)) is \( \sinh^2 (\gamma |A_{pu}| r) \), that coincides with \( n_{PDC} = \sinh^2 (\kappa) \) upon identification of \( \kappa = \gamma |A_{pu}| r \).

We can now use Eq. (23) to calculate the value of \( |\Delta \mathbf{k}|^2 \) corresponding to the full width at half maximum (FWHM) of the gain

\[
|\Delta \mathbf{k}|_{\text{FWHM}}^2 = \frac{4 \ln 2\gamma |A_{pu}|}{r \cos^2 (\psi_{si,id}/2)} (\equiv D). \tag{24}
\]

Since \( |\Delta \mathbf{k}|^2 \) is a function of the frequencies and of the angles of the parametric emission, we can expand it about the phase-matching condition and find the corresponding values of angular and temporal bandwidths.

First of all, we consider the angles fixed at their values in phase matching and expand \( |\Delta \mathbf{k}|^2 \) in a Taylor’s series up to the second order of the expansion of the wavevectors about \( \bar{\omega}_{si} \) and \( \bar{\omega}_{id} \), \( \bar{\omega}_j \) being the central frequency of signal and idler fields. By assuming frequency- and phase-matching for the central frequencies of the interacting fields \( (\bar{\omega}_{pu} = \bar{\omega}_{si} + \bar{\omega}_{id} \text{ and } \bar{k}_{pu} / \bar{\omega}_{pu} = \bar{k}_{si} / \bar{\omega}_{si} - \bar{k}_{id} / \bar{\omega}_{id} = 0) \), we get \( \delta \omega_{si} = -\delta \omega_{id} (\equiv \delta \omega) \) and

\[
|\Delta \mathbf{k}|^2 \approx \frac{1}{v_{g,si}^2} + \frac{1}{v_{g,id}^2} - \frac{2}{v_{g,si} v_{g,id}} \cos \left( \psi_{si,id} \right) (\delta \omega)^2 (\equiv \mathcal{A}(\delta \omega)^2), \tag{25}
\]

where \( v_{g,j} = [(\partial k / \partial \omega)_{\bar{\omega}_j}]^{-1} \) is the group velocity. By jointly using Eqs. (24) and (25) we obtain

\[
\Delta \omega_{\text{FWHM}} = 2 \sqrt{D/A}, \tag{26}
\]

and the number of temporal modes

\[
\mu \equiv \frac{\tau}{\Delta t_{\text{FWHM}}} = \frac{\tau \Delta \omega_{\text{FWHM}}}{4 \ln 2}. \tag{27}
\]
By following a similar reasoning, we calculate the dependence of \(|\Delta k|^2\) on the angles \(\theta\) and \(\beta\) defined in Fig. 1:

\[
|\Delta k|^2 \simeq B\delta \beta^2 + T\delta \theta^2,
\]

in which:

\[
B = k_s^2 \left\{ 1 + \frac{\cos^2 \theta_{si}}{\cos^2 \theta_{id}} - \frac{2 \cos \theta_{si} \cos \theta_{id}}{\cos \theta_{id}} \left[ \cos \beta_{si} \cos \beta_{id} + \sin \beta_{si} \sin \beta_{id} \cos (\theta_{si} - \theta_{id}) \right] \right\},
\]

\[
T = k_s^2 \left\{ \cos^2 \beta_{si} + \cos^2 \beta_{id} - 2 \cos \beta_{si} \cos \beta_{id} \cos (\theta_{si} - \theta_{id}) \right\}.
\]

The expansion in Eq. (28) can be expressed in terms of \(\delta \theta\) or \(\delta \beta\) only using the relations given by phase matching. By jointly exploiting Eqs. (24) and (28) we obtain

\[
\Delta \theta_{si, \text{FWHM}} = 2\sqrt{D/(B + \tan^2 \theta_{si}/\tan^2 \beta_{si} T)}
\]

\[
\Delta \beta_{si, \text{FWHM}} = 2\sqrt{D/(T + \tan^2 \beta_{si}/\tan^2 \theta_{si} T)},
\]

that can be compared with the experimental results.

Note that the above discussion relies on the parametric approximation, that is, in the case of nonevolving pump beam. On the contrary, at increasing values of pump power the correlation widths in spectrum and space start increasing monotonically and then decrease at higher powers due to the onset of pump depletion (Allevi et al., 2014a). This can be explained with the different evolution of various modes describing the generated field. The best description to account for this evolution is that of the Schmidt modes (Peñina, 2016).

### 2.4 Schmidt Modes

The Schmidt modes represent a second approach to determine the number of modes in the PDC output. For TWB in the regime of single pairs, the description with Schmidt modes is well established and gives the biphoton function, which is expressed as a sum of factorized terms (Christ et al., 2011; Just et al., 2013; Law and Eberly, 2004)

\[
|\psi\rangle = \sum_k \lambda_k |u_k\rangle |v_k\rangle.
\]

Here \(|u_k\rangle\) and \(|v_k\rangle\) represent the eigenvectors of the orthonormal dual basis of the Schmidt modes. The eigenvalues \(\lambda_k\) of the decomposition give the
probabilities $p_k$ of detecting a photon in the $k$-th mode, $p_k = \lambda_k^2$. As a more synthetic characterization of the state, we can define the Schmidt number

$$K = \sum_k \frac{1}{\lambda_k^2}. \quad (33)$$

In contrast to low-gain PDC, developing a consistent theoretical description of high-gain TWBs is a difficult problem due to the contribution of correlated high-order Fock components, which prevents the application of the perturbation theory. As shown below, a way to approach the Schmidt decomposition of high-gain TWB is to define polychromatic modes that allow a formally similar description as in the low-gain regime as presented by Peřina (2016), Sharapova et al. (2015), and Perina et al. (2016).

At the single-photon level, the Schmidt decomposition, and in particular the number $K$ of modes, can be used to assess the degree of entanglement of the TWB states (Law and Eberly, 2004). To the same aim, we can also introduce a quantifier parameter $F$, usually called Fedorov ratio, which is defined as the ratio of widths of single-particle ($\Delta p$) and coincidence wave packets ($\delta p$) (Mikhailova et al., 2008)

$$F = \frac{\Delta p}{\delta p} = \frac{\Delta q}{\delta q}, \quad (34)$$

being $p$ and $q$ the transverse wavevectors of signal and idler, respectively. Fedorov et al. (2006) demonstrated that, for double-Gaussian bipartite states, the parameter $F$ coincides with the Schmidt number $K$, defined as the inverse of the purity of the state of each separate subsystem

$$K = \frac{1}{Tr[\rho_1^2]}. \quad (35)$$

Moreover, Mikhailova et al. (2008) have demonstrated that in more realistic cases, in which the biphoton function is not double-Gaussian, the parameters $F$ and $K$ are very close to each other.

The value of the Fedorov ratio can be rather easily evaluated from coincidence measurements, while the Schmidt number can be obtained from the theory of the mode decomposition, in which the biphoton function is expressed as a sum of factorized terms (Christ et al., 2011; Just et al., 2013; Law and Eberly, 2004).
In the case of more intense TWB, we can define a quantity $F_\theta$ in analogy to Eq. (34) by taking the ratio of the angular size of the downconverted light to the angular size of the cross-correlation area:

$$F_\theta = \frac{\Delta \theta}{\delta \theta}.$$  \hspace{1cm} (36)

For what concerns $K$, a quantity analogous to the Schmidt number can be evaluated by calculating the second-order intensity autocorrelation (Allevi and Bondani, 2014; Allevi et al., 2014a; Mauerer et al., 2009; Pérez et al., 2014). In particular, for intense TWBs the probability $p_k$ of detecting a photon in the k-th mode can be derived from the mean intensity $\langle I_k \rangle$ of, e.g., the signal field along the formula $p_k = \langle I_k \rangle / \langle I \rangle$, in which $I = \sum_k I_k$ is the overall intensity. An effective number of populated spatio-spectral modes $K$ is then determined as (Gatti et al., 2012; Peřina, 2013)

$$K = \frac{1}{\sum_k p_k^2}.$$  \hspace{1cm} (37)

Assuming thermal statistics of individual modes $k$ generated at high-gain PDC (even with pump depletion) (Allevi and Bondani, 2014), the relation $\langle I_k^2 \rangle = 2 \langle I_k \rangle^2$ holds, as shown by Perina (1991). Then Eq. (37) can be rewritten into the form:

$$K = \frac{\left( \sum_k \langle I_k \rangle \right)^2}{\sum_k \langle I_k \rangle^2} = \frac{\left( \sum_k \langle I_k \rangle \right)^2}{\sum_k \left( \langle I_k^2 \rangle - \langle I_k \rangle^2 \right)}.$$  \hspace{1cm} (38)

On the other hand, the intensity autocorrelation coefficient defined as $g^{11} = \langle I^2 \rangle / \langle I \rangle^2$ (Perina, 1991) is expressed as

$$g^{11} = 1 + \frac{\sum_k \left( \langle I_k^2 \rangle - \langle I_k \rangle^2 \right)}{\left( \sum_k \langle I_k \rangle \right)^2}.$$  \hspace{1cm} (39)

The comparison of Eqs. (38) and (39) finally provides the formula

$$g^{11} = 1 + \frac{1}{K}.$$  \hspace{1cm} (40)
demonstrating the connection to the Schmidt number. This equation can thus be used for the determination of the number $K$ of modes.

We notice that in the high-gain regime there is also a relevant quantum correlation between the signal and idler photon numbers inside independent spatio–spectral modes. For this reason, the evaluation of the number of spatio–spectral modes is not sufficient to determine the quantum Schmidt number quantifying the entanglement of the state and further considerations are needed.

3. EXPERIMENTS

3.1 Introduction to the Different Regimes and Detectors

In order to characterize the properties of TWB states, many experimental works have been performed till now in very different intensity regimes and with different detection schemes. In principle, there are two different kinds of detection strategies, namely, the optical homodyne tomography (OHT) (Lvovsky and Raymer, 2009) and the direct detection. The two techniques are complementary: OHT gives access to wave-like properties of light (quadrature distributions), whereas direct detection is aimed at studying particle-like features (number of photons).

From the technical point of view, OHT is based on an interferometric scheme in which the state to be characterized is mixed with a high-intensity coherent state with variable phase called “local oscillator” (LO). The two outputs of the interferometer are detected by two pin photodiodes, whose difference photocurrent is suitably amplified and recorded as a function of the LO phase. By properly processing the data and applying a reconstruction algorithm, it is possible to retrieve the complete knowledge of the state under study. As the TWB state is formally a bipartite entangled state, its characterization by means of OHT would require a double homodyning system, in order to simultaneously investigate signal and idler. This makes the strategy quite challenging, even if some experimental works on double homodyne have been performed over the years (D’Ariano et al., 1998).

On the contrary, direct detection is more straightforward, even if a suitable modeling of the detection apparatus is needed to extract the proper information about the state under study. Many direct-detection schemes have been implemented to investigate not only the photon–number statistics of a single arm of TWB (Allevi and Bondani, 2014; Avenhaus et al., 2008; Dovrat et al., 2012; Kalashnikov et al., 2011; Mauerer et al., 2009; Paleari et al., 2004; Peřina et al., 2012; Waks et al., 2006; Wasilewski et al., 2008),
but also the photon-number correlations between signal and idler (Allevi et al., 2012; Avenhaus et al., 2010; Bartley et al., 2013; Dovrat et al., 2013; Harder et al., 2016; Kalashnikov et al., 2012). To this aim, most investigations were produced in the single-photon regime by means of single-photon detectors, such as single-photon avalanche diodes (SPADs), photomultipliers and intensified CCD (iCCD) cameras, or electron multiplying CCD (EMCCD) cameras (Migdall et al., 2013). In the opposite intensity domain, multi-photon TWB states have been investigated by means of CCD cameras, operated either at room temperature or with a proper cooling (Brida et al., 2009a; Jedrkiewicz et al., 2004), or of pin photodiodes (Agafonov et al., 2010; Bondani et al., 2007). In the last two decades also TWB states generated in the intermediate regime, i.e., the so-called mesoscopic domain, have been measured and characterized by means of different classes of photon-number resolving detectors (Allevi et al., 2010; Lamperti et al., 2014). Among them, it is worth mentioning the hybrid photodetector (HPD), which combines a photocathode with a diode structure operated below the breakdown threshold (Andreoni and Bondani, 2009; Bondani et al., 2009a). At variance with the traditional photomultipliers, in such a detector the amplification process occurs in a single step, so that the excess noise is small enough to allow photon-number resolution (up to six detected photons). The quantum efficiency of the detector, mainly given by the photocathode, can reach good values (up to 0.5 in the visible range), whereas the dark-count rate can be neglected. As an alternative, the accomplishment of photon-counting capability can be obtained by splitting the light to be measured either in space or in time prior to detection so that at most one photon at a time hits the detector sensitive area. Among these detectors, we mention the visible light photon counter (VLPC) (Kim et al., 1999); the fiber-loop detector, which is a time-multiplexed detector based on one (Řeháček et al., 2003) or more (Fitch et al., 2003) SPADs; the silicon photomultiplier (SiPM), that is, constituted by a matrix of SPADs with a common output (Akindinov et al., 1997). Due to their composite structure, SiPMs have a good photon-counting capability, even if their large dark-count rate (100 kHz in the new generation) and their not negligible cross-talk probability (Afek et al., 2009; Ramilli et al., 2010) have till now prevented their use in the investigation of the nonclassical character of TWB states, except at very low-intensity levels. In the last decade, a tremendous progress has been achieved in the field of superconductors, so that new types of detectors have been developed, such as the transition-edge sensors (TES) (Lita et al., 2008) and the superconducting nanowires (Gol’tsman
et al., 2001). Despite having a good quantum efficiency, these detectors must operate at cryogenic temperatures and thus their operation is rather cumbersome. As of today, the ideal detector has yet to appear and the optimal choice is application specific.

For what concerns the characterization of TWB states, it is worth noting that most of the experimental works were conducted in a limited intensity regime by using a specific kind of detector. Nevertheless, TWB states could in principle change their properties (mean number of photons, size of the spectral and spatial areas, photon-number correlation coefficient, number of Schmidt modes, etc.) as a function of pump power and a full investigation would be necessary. Indeed, a complete investigation of the nature of TWBs at different PDC gains has received only limited attention due to the difficulty of finding a proper detection apparatus able to cover a sufficiently wide intensity range. For instance, iCCD and EMCCD cameras can be in principle operated both at single-photon level and in the macroscopic domain. However, they cannot be used in the transient region, where the noise due to the spread of the gain strongly limits the signal-to-noise ratio. On the contrary, in the following Section 3.3 we show a different strategy that we recently adopted to investigate the statistical properties of TWBs in different intensity regimes.

### 3.2 The Laser System

All the experimental results we will discuss in the following were obtained by exploiting a few-ps-pulsed mode-locked Nd:YLF laser (High-Q Laser, Austria), regeneratively amplified at 500 Hz, as the laser source. The emitted fundamental wavelength is at 1047 nm, and second and third harmonics are built in. This source produces pulses having high-peak intensities (up to some GW/cm$^2$ at 349 nm) and a quite narrow spectral bandwidth (less than 1 nm at 349 nm). Such features make it possible the generation of bright pulsed TWBs, thus allowing the investigation of the PDC process at different intensity levels. In fact, the ps-pulses are at the same time less damaging than ns-pulses of the same intensities and narrower in spectrum than fs-pulses. As to the pump field, most of the results presented in this review have been obtained with the third harmonics of the laser at 349 nm. Frequency-degenerate TWB states at 698 nm can still be detected by cameras and photon-number resolving detectors operating in the visible range. In order to match the maximum quantum efficiency of the detectors, we also used the fourth harmonics at 262 nm, which allows the generation of TWBs
at frequency degeneracy in the green spectral region (at 523 nm). The only drawback in this case is that the intensity of each laser shot is not enough to produce high-intensity TWBs. For this reason, the fourth harmonics is particularly useful to investigate TWBs in the mesoscopic domain, as shown in Sections 3.5 and 4. Finally, we remark that the repetition rate of the laser source represents a good compromise between the amount of data required to study the statistical properties of light and the operation rate of the detectors. In particular, 500-Hz-repetition rate is the upper limit for the good operation of the EMCCD camera used to characterize the spatio-spectral properties of TWB (see Section 3.4).

3.3 Photon-Number Statistics

As stated in Section 2, the statistics of photons in each arm of TWB is multimode thermal. This kind of distribution has been widely measured in different intensity regimes and with different kinds of detectors. Here we present a different approach, that is, the investigation of the statistical properties at different intensity levels by means of the same detection apparatus. The detection scheme is essentially based on the use of neutral-density filters and of a specific class of PNR detectors, namely, HPDs (Allevi and Bondani, 2014). As mentioned earlier, such detectors have negligible dark-count rate and after-pulse probability. The main drawback is represented by the limited detection efficiency, which is actually much lower than the value given by the manufacturer for the photocathode due to the non-perfect photon-number resolution. However, in the present case such a feature does not constitute a limitation, thanks to the invariance of the multimode thermal statistics under attenuation.

3.3.1 Experimental Setup

The experimental apparatus used to investigate the statistical properties is shown in Fig. 3. The laser output, collimated by a telescope, was sent to a type-I 8 mm long $\beta$-barium borate (BBO) crystal (cut angle = 37 degree). The crystal was tuned to have the phase-matching condition at frequency degeneracy in quasi-collinear configuration (33.8 degree). The full width at half maximum of the pump beam was $\sim$370 $\mu$m at the lowest pump power. Two twin portions of PDC light were passed through the vertical slit of an imaging spectrometer (Lot Oriel) with a 600 lines/mm grating inside, which is a device that gives access to the joint experimental investigation of spatial (in angle) and spectral features of the input light. The slit of
the spectrometer was located at 40 cm from the nonlinear crystal to approach the far-field condition without using lenses.

A multimode fiber having a core diameter of 300 μm was located at ~10 cm from the exit plane of the device in order to roughly collect a single spatio-spectral area in one arm of the TWB. The fiber was mounted on a three-axis translation stage. The light was then delivered to a hybrid photodetector (HPD, mod. R10467U-40, Hamamatsu, nominal quantum efficiency 30% at 698 nm and 25% at 349 nm), whose output was amplified (preamplifier A250 plus amplifier A275, Amptek), synchronously integrated (SGI, SR250, Stanford), and digitized (ADC, PCI-6251, National Instruments).

As better explained in Appendix, to analyze the output of this detection chain, which is expressed in voltage units, the detection process is modeled in two steps: a Bernoullian convolution and an overall amplification/conversion process given by a very precise constant factor, γ. We notice that, as explained in Appendix, the factor γ can be obtained from the measurement of the same light at different values of the overall detection efficiency.

Fig. 3 Sketch of the experimental setup. ADC+PC, analog-to-digital converter; BBO, nonlinear crystal; CCD, CCD camera; F, colored glass filters; G, grating; HPD, hybrid photodetector; HWP, half-wave plate; L, lens; M_j, spherical mirrors; M, UV mirror; MF, multimode fiber (300 μm core diameter); ND, removable neutral-density filters; PBS, polarizing cube beam splitter; SGI, synchronous gated integrator. Adapted from Allevi, A., Bondani, M., 2014. Statistics of twin-beam states by photon-number resolving detectors up to pump depletion. J. Opt. Soc. Am. B 31 (10), B14–B19. http://dx.doi.org/10.1364/JOSAB.31.000B14.
of the apparatus. In fact, it has been demonstrated in (Bondani et al., 2009a) that the Fano factor $F_v = \sigma_v^2/\langle v \rangle$ for the output voltages is a linear function of the mean value of the output voltages. In particular, the limit of this function for the mean values approaching zero gives the value of $\gamma$. In the case of HPD, such a value also coincides with the distance between two consecutive valleys in the pulse-height spectrum of the detector output. The number of detected photons is thus obtained by the following procedure: each output voltage is subtracted of the mean value of the electronic noise measured in the absence of light, then the resulting values are divided by the value of $\gamma$, determined in one of the two ways described above, and rebinned in unitary bins. To perform a systematic characterization of the optical states under investigation, a proper data sample of consecutive laser pulses is usually acquired, thus allowing the study of the statistical properties of light. In the present case, for each choice of the pump power each experimental run was repeated 100,000 times. A set of neutral-density filters was used to suitably attenuate the light and keep it within the detector dynamics. For the data presented here we used filters from 0 to 6 optical densities, whose transmittance was measured at the specific wavelength. In such a way we could measure a signal beam in the range from $0.6$ to $1.75 \times 10^6$ mean detected photons. The errors of the various quantities plotted in the following were calculated as the standard deviations of their mean values evaluated for blocks of 20,000 consecutive shots. The power was changed by means of a half-wave plate, followed by a polarizing cube beam splitter located in front of the BBO crystal, in order to keep the same spatial profile of the pump.

### 3.3.2 Results

In Fig. 4A we plot the experimental detected-photon distribution (dots) corresponding to different choices of pump power (black: 18.7 mW, red: 22.7 mW, green: 27 mW, blue: 58.9 mW, and magenta: 89 mW). As anticipated above, the neutral-density filters kept the light value within the dynamic range of the HPD detectors. For this reason, there is no direct connection between the pump power values and the measured mean values of the TWB. Since the light measured on either signal or idler separately is classical, the presence of the filters does not change the photon-number distribution and, in particular, does not affect the number of independent modes. Fig. 4A reports also the multimode thermal fitting curves (solid lines) for the experimental data obtained by exploiting Eq. (11), in which the mean value, $\langle m \rangle$, is fixed to the values experimentally measured and the number of modes, $\mu$, is the only fitting parameter. The $\mu$ values obtained from those
fits are plotted in Fig. 5. The very good agreement between data and theory is attested by the very high values of the fidelity shown in Fig. 4B (full circles). As a comparison, in Fig. 4A, we also plot the Poissonian curves (dashed lines) calculated for the experimental mean values. As shown by the fidelity values in Fig. 4B (empty circles), the superposition to the

Fig. 4  (A) Examples of experimental detected-photon number distributions (dots), multimode-thermal fit (solid lines), and Poissonian fit (dashed lines). Results obtained at different pump powers: 8.7 mW in black, 22.7 mW in red, 27 mW in green, 58.9 mW in blue, and 89 mW in magenta. (B) Fidelity for the fit to a multimode-thermal distribution (full dots) and fidelity for the fit to a Poissonian distribution (empty circles). Adapted from Allevi, A., Bondani, M., 2014. Statistics of twin-beam states by photon-number resolving detectors up to pump depletion. J. Opt. Soc. Am. B 31 (10), B14–B19. http://dx.doi.org/10.1364/JOSAB.31.000B14.

Fig. 5 Number $\mu$ of spatio-spectral modes as a function of the pump mean power calculated from the statistics of the number of photons detected in the signal arm (black dots). Adapted from Allevi, A., Bondani, M., 2014. Statistics of twin-beam states by photon-number resolving detectors up to pump depletion. J. Opt. Soc. Am. B 31 (10), B14–B19. http://dx.doi.org/10.1364/JOSAB.31.000B14.
experimental data is definitely worse. We conclude that the multimode thermal nature of the photon-number statistics does not depend on pump power. On the contrary, the number of modes is strongly influenced by the complex evolution of the system, as it appears from the data of Fig. 5. In particular, it is worth noting the appearance of a minimum in the evolution of $\mu$ as a function of the input power, which appears for a specific value of the pump mean value. In order to understand what really happens inside the crystal, it is instructive to also consider the evolution of the mean number of detected photons in one of the TWB arm as a function of the pump mean power.

In Fig. 6 we plot the mean number of detected photons in the signal arm as a function of the pump mean power. The plotted values were calculated by taking into account the transmittance of the filters. First of all, we observe that for low pump power values the mean number of photons increases exponentially, as expected for high-gain PDC under the hypothesis of undepleted pump beam. This initial behavior is emphasized in the inset of the picture, where the experimental data corresponding to the lowest pump power values are presented. In the main figure, we see that the occurrence of a progressive pump depletion at very high-gain PDC processes prevents the exponential growth of the mean number of photons. On the

![Graph of mean number of photons detected in the signal arm as a function of pump mean power.](image)

**Fig. 6** Evolution of the mean number of photons detected in the signal arm as a function of the pump mean power. Inset: first part of the data shown in the main figure exhibiting an exponential growth according to undepleted pump approximation. Adapted from Allevi, A., Bondani, M., 2014. Statistics of twin-beam states by photon-number resolving detectors up to pump depletion. J. Opt. Soc. Am. B 31 (10), B14–B19. [http://dx.doi.org/10.1364/JOSAB.31.000B14](http://dx.doi.org/10.1364/JOSAB.31.000B14).
contrary, a linear dependence on the pump power begins. We notice that the change in the slope roughly occurs at the value of pump mean power, where the minimum in the number of modes of Fig. 5 occurs.

3.4 Spatio-Spectral Properties

The evolution of the statistical properties of TWB in terms of photon-numbers is strictly connected to the evolution of spatial and spectral properties. Indeed, it is well known that TWB exhibits spatial and spectral correlations. As already anticipated in Section 1, the first experimental measurements of spatial correlations in TWB, aimed at determining the size of the coherence areas in a transverse plane, were performed at single-photon level on TWB states by means of scanned single-photon detectors (Blanchet et al., 2008; Haderka et al., 2005b; Hamar et al., 2010). The experimental results can be compared to a well-established theory properly working in this regime, in which, for instance, the pump beam can be considered as nonevolving during the interaction (undepleted pump approximation). On the other hand, TWB can also be generated in a much higher intensity regime (Agafonov et al., 2010; Bondani et al., 2007; Brida et al., 2009a; Jedrkiewicz et al., 2004), in which coherence areas become visible in single-shot images taken at the output of the nonlinear crystal. More recently, also the spectral features of macroscopic TWBs have been investigated in the collinear interaction geometry close to frequency degeneracy (Spasibko et al., 2012). Moreover, in the high-gain regime, the X-shaped coherence of the PDC output field (Spasibko et al., 2012) and the X-shaped spatiotemporal TWB near-field correlations (Gatti et al., 2009; Jedrkiewicz et al., 2006, 2012), originating from the space–time coupling in the phase matching, have been demonstrated.

Here, on the base of Allevi et al. (2014a), we show that the use of an EMCCD camera, combined to an imaging spectrometer that resolves emission angles and wavelengths simultaneously, represents an interesting solution to investigate the evolution of spatial and spectral properties in the high-gain regime, where pump depletion can suddenly occur (Allevi et al., 2014a).

The experimental setup used for the measurement of the PDC light structure in the angular and spectral (θ, λ) domain is shown in Fig. 7A. A type-I 8 mm-long BBO crystal (cut angle = 37 degree) was pumped by the third-harmonic pulses of the laser source described in Section 3.2. The FWHM of the pump beam, collimated by means of a telescope in front of the BBO, was \(\sim 380 \, \mu \text{m}\) at the lowest pump power. Indeed, the pump mean power was
changed during the experiment by a half-wave plate followed by a polarizing cube beam splitter. The crystal was tuned to have phase-matching at frequency degeneracy in slightly noncollinear configuration. The broadband PDC light was collected by a 60-mm focal length lens and focused on the plane of the vertical slit of the already mentioned imaging spectrometer. The angularly dispersed far-field radiation was then recorded in single shot by a synchronized EMCCD camera (iXon Ultra 897, Andor), operated at full frame resolution (512 × 512 pixels, 16-μm pixel size). The resulting resolution of the system composed of the imaging spectrometer and the EMCCD camera was 0.2 nm in spectrum and 0.015 degree in angle. A typical speckle-like pattern is shown in Fig. 7B, in which the horizontal axis is connected to spectrum and the vertical axis to the angular dispersion.

The existence of intensity correlations between the signal and idler portions of the TWB is well supported by the presence of symmetrical speckles around the degenerate wavelength and the collinear direction.

Fig. 7 (A) Experimental setup used for the spatio-spectral measurements of the TWB. BBO, nonlinear crystal; EMCCD, electron multiplying camera; G, grating; HWP, half-wave plate; PBS, polarizing cube beam splitter. (B) Single-shot image recorded by the EMCCD camera, in which the typical speckle-like pattern of PDC in the spatio-spectral domain is clearly evident. (C) Typical contour plot of the correlation matrix $\Gamma$, in which the single pixel $(i, j)$ is chosen on the left side. The intensity autocorrelation area is on the same side, whereas the cross-correlation area is on the right side. Adapted from Allevi, A., Jedrkiewicz, O., Brambilla, E., Gatti, A., Perina, J., Haderka, O., Bondani, M., 2014a. Coherence properties of high-gain twin beams. Phys. Rev. A 90, 063812. http://dx.doi.org/10.1103/PhysRevA.90.063812; Perina Jr., J., Haderka, O., Allevi, A., Bondani, M., 2016. Internal dynamics of intense twin beams and their coherence. Sci. Rep. 6. http://dx.doi.org/10.1038/srep22320.
The evolution of the patterns at different pump mean powers $P$, and hence at different PDC gains, was investigated by calculating the intensity correlation coefficient between a single pixel at coordinates $(i, j)$ and all the pixels $(k, l)$ contained in a single image

$$
\Gamma_{k,l}^{(i,j)} = \frac{\langle I_{i,j}I_{k,l} \rangle}{\langle I_{i,j} \rangle \langle I_{k,l} \rangle},
$$

where $I$ is the intensity value of each pixel expressed in digital numbers and upon subtraction of the mean value of the noise measured with the camera in perfect dark, whereas $\langle \ldots \rangle$ indicates the averaging over a sequence of 1000 subsequent images. The procedure was applied to a set of pixels having the abscissa $i$ close to frequency degeneracy and the ordinate $j$ in the quasi-collinear direction.

The function $\Gamma_{k,l}^{(i,j)}$ defined in Eq. (41) is a matrix having the same size as the original images, in which the horizontal axis, running on the index $k$, is connected to spectrum, while the vertical axis, running on the index $l$, is connected to the angular dispersion. As an example, in Fig. 7C the contour plot of the correlation matrix, obtained by choosing the pixel at coordinates $(i, j)$ on the left side of the images, is shown. The intensity autocorrelation area is on the same side, whereas the cross-correlation area is on the right side.

Fig. 8 shows the behaviors of the spectral (panel A) and spatial, i.e., in angular domain, (panel B) widths, FWHM, of the intensity autocorrelation and cross-correlation areas, as functions of the input pump mean power. In both panels, we can observe an initial growth that reaches the maximum at a pump power of about 30 mW and then decreases. As shown in the figure, only the first part of the data is well described by a fourth-root square function of pump power. This kind of evolution is predicted by the theory of coherence areas under the assumption of undepleted pump beam (Bondani et al., 2012; Brambilla et al., 2004; Brida et al., 2009a). The second part of our experimental results (including the peak and the decrease in the FWHM) clearly indicates that for high pump power values the assumption of undepleted pump beam does not hold anymore. Indeed, in this situation also the pump beam evolves nontrivially and the corresponding equations of motion for the three-mode interaction can be solved only numerically. Indeed, the behavior shown in Fig. 8 can be qualitatively reproduced by means of a numerical simulation of the PDC process, as already shown in Allevi et al. (2014a). In the numerical modeling the propagation equations
that describe the PDC process generated from vacuum fluctuations are solved through a pseudo-spectral (split-step) integration method. The PDC field at the crystal entrance face, which is initially in the vacuum state, is simulated with a Gaussian white noise in the framework of the Wigner representation, while the injected pump field is treated as a classical coherent pulse with a transverse FWHM size of 250 μm and a pulse duration of 400 fs (more details can be found in Brambilla et al. (2004)). The modeling takes into account both the spatial and the temporal degrees of freedom of the system (three spatial dimensions + time). The phase-matching conditions in the BBO crystal are described by using the complete Sellmeier dispersion relations found in Boeuf et al. (2000). The results of these numerical simulations qualitatively reproduce the experimental data. The slight discrepancy between the absolute values of the experimental FWHMs and those obtained from simulations is mainly due to the uncertainty in the correct positioning of the EMCCD camera in the imaging exit plane of the spectrometer.

A further confirmation of the occurrence of pump depletion is given by a specific experimental investigation of the spectral and spatial pump-beam profiles shown in Fig. 9. From the experimental point of view, we obtained the spectral profile of the pump by producing a magnified image of the near field of the pump on the slit of the spectrometer (in this case we employed a
grating characterized by 2400 lines/mm) and using a CCD camera (DCU223M, Thorlabs, 1024 × 768 pixels, 4.65-μm pixel size) to collect the light at the output. Fig. 9A displays different sections, normalized at their peaks, corresponding to different pump mean power values. First of all, we observe that the spectrum of the pump turns out to be roughly ~1 nm wide, thus testifying that the pump beam is nontransform-limited and also justifying the choice of a 400-fs pulse duration in the simulations. Secondly, we note that both the dips in the sections of panel (A) and the appearance of a central hole in the contour plot shown in panel (C) are a clear signature of pump depletion. The sections of the spatial profiles presented in Fig. 9B and normalized at their areas were obtained by taking 1:1 images of the pump beam at the output of the crystal with the same DCU223M camera at different values of the power. Also in this case a clear dip occurs. It becomes broader and deeper as the pump mean power increases. Its generation is

**Fig. 9** Pump beam profiles, spectral and spatial in (A) and (B), respectively, for different values of the pump mean power. Green: 15 mW, magenta: 35 mW, blue: 55 mW, and black: 99 mW. Map of the spectral distribution (in panel C) and of the spatial distribution (in panel D), both taken at 55 mW upon subtraction of the distribution of the least intense measurement. Adapted from Allevi, A., Jedrkiewicz, O., Brambilla, E., Gatti, A., Perina, J., Haderka, O., Bondani, M., 2014a. Coherence properties of high-gain twin beams. Phys. Rev. A 90, 063812. [http://dx.doi.org/10.1103/PhysRevA.90.063812](http://dx.doi.org/10.1103/PhysRevA.90.063812).
initially slightly lateral with respect to the center because of the pump beam walk-off inside the crystal, as shown in the bottom panel (D).

### 3.4.1 Far-Field Coherence Areas

As already described in Section 2, the evolution of the size of intensity auto- and cross-correlation areas, due to pump depletion, can be explained in terms of the modes that describe the radiation field. When the PDC process occurs at gain values leading to depletion, also the pump beam evolves in the nonlinear interaction and the dynamics of the system becomes more complex. In particular, there is a dependence of the number of effectively populated signal and idler radiation modes on the pump power. As the pump power increases, the PDC gain profile becomes narrower and narrower, and thus signal and idler fields are dominantly emitted into a smaller and smaller number of modes that gain energy to the detriment of the others (Pérez et al., 2014; Wasilewski et al., 2006). For sufficiently high values of the pump power, the process of mode selection reverts as the pump profile undergoes depletion. For this reason, the gain of the high-populated low-order modes is on the one side reduced, whereas the gain of low-populated higher-order modes is on the other side supported. Such a behavior explains the narrowing of the intensity auto- and cross-correlation areas shown in Fig. 8. The description in terms of populated radiation modes also explains the slight discrepancy between auto- and cross-correlation intensity functions plotted in Fig. 8. In fact, the cross-correlation function reflects the mutual coherence between signal and idler and originates in the pairwise PDC emission, whereas the autocorrelation function expresses the internal coherence due to the presence of three evolving fields. As such, it is more sensitive to losses in the mode selection.

With the goal of investigating how the variation of the coherence area is affected by a variation in the number of effectively populated modes, we evaluated the peak of the intensity correlation as a function of the pump power. More precisely, we evaluated the autocorrelation coefficient in Eq. (41) over a single pixel, $\Gamma_{i,j}^{(i,j)}$, so that we can exploit the best resolution of our detection system, namely 0.2 nm in spectrum and 0.015 degree in angle (Laiho et al., 2011). The procedure was applied to a set of pixels close to the frequency degeneracy and in a nearly collinear direction. In Fig. 10A we present the dependence of the maximum values of intensity autocorrelation coefficient on increasing values of the pump mean power. As already observed in Fig. 8 for the size of intensity auto- and cross-correlation areas,
the plot exhibits a peak for the pump power of 30 mW that lies at the beginning of the pump depletion regime. The number $K$ of modes is obtained according to Eq. (40), which formally represents the macroscopic analogous of the Schmidt number. The experimental values of $K$ are shown as green dots in Fig. 10B. The comparison between this panel and the plots in Fig. 8 confirms the complementary behavior for the size of the intensity auto- and cross-correlation areas and the numbers of modes. Note also that the behavior of $K$ is really similar to what we found for the evolution of the number of modes from statistics as presented in Section 3.3.

The evolution of the system at different pump powers can be alternatively investigated in terms of the quantity $F_\theta$ defined by Eq. (36). As already explained in Section 2, such a quantity, as well as its spectral counterpart, represents the analogous at high-intensity level of the Fedorov ratio, defined by Eq. (34) and commonly used to quantify the entanglement degree of the TWB state at single-photon level.

In order to calculate the values of $F_\theta$, first of all we experimentally determined the behavior of the spatial bandwidth of the PDC as a function of pump power (Allevi et al., 2015). The bandwidth was calculated as the FWHM of the vertical section of the average far-field spectrum in correspondence to a number of different pixels close to frequency degeneracy. The second quantity required to evaluate $F_\theta$ is the size of

the cross-correlation area, already shown in Fig. 8. In Fig. 10B we plot the value of $F_\vartheta$ (black dots) as a function of the pump mean power. The behavior of the experimental data shows a trend with a minimum corresponding to the onset of pump depletion. From a direct comparison between $K$ and $F_\vartheta$, we notice that the two quantities are not perfectly equal as expected from the theory in the single-photon regime and for double-Gaussian biphoton function. Nevertheless, they display the same trend and a nearly proportional behavior.

### 3.4.2 Transition From Near-Field to Far-Field Coherence Areas

The results presented above were obtained in the far-field configuration. In order to get a more complete knowledge of the system, the investigation of the spatio-spectral properties of TWB in the transient region from near to far field is also interesting. In this context, most investigations have been performed in the spatial domain, either in the near-field or in the far-field configurations (D’Angelo et al., 2004; Hamar et al., 2010; Howell et al., 2004), as well as in both (Edgar et al., 2012). In addition, attention has been devoted also to the transient area between the two extremes (Chan et al., 2007; Dyakonov et al., 2015; Just et al., 2013; Tasca et al., 2009). While in the near-field position intensity cross-correlations have been confirmed in Edgar et al. (2012), momentum anticorrelations have been observed in the far field (Edgar et al., 2012; Hamar et al., 2010), as a result of the phase-matching conditions. In the transient regime, cross-correlations get blurred and, at a certain position, they cannot be observed at all (Chan et al., 2007). At this position, entanglement is entirely transferred to the phase of the two-photon amplitude (Chan et al., 2007), thus becoming hidden to intensity observations. The evolution of cross-correlations from near field to far field has been experimentally observed at the single-photon level (Just et al., 2013) and the Fedorov ratio (Fedorov et al., 2005) has been determined. In the experiment of Just et al. (2013), the Fedorov ratio equal to 1 has been measured at the position where cross-correlations were spread over the whole PDC beam. We also note that nearly all these investigations were carried out in the single-photon regime, in which a well-established theory based on the biphoton function exists. Here, we summarize the results recently published in Haderka et al. (2015), where we experimentally demonstrated the physical mechanism ruling the behavior of both spatial and spectral autocorrelations and cross-correlations of macroscopic TWB in the transition from the near field to the far field.
The experimental setup is depicted in Fig. 11. The third harmonics of the already described Nd:YLF laser was used to pump a type-I BBO crystal (4 mm long, cut $\Theta = 33.8$ degree) in a nearly collinear configuration and placed on a roto-translation stage. First of all, the output face of the crystal was precisely set (distances $a,a'$ in Fig. 11) to be imaged to the plane of the input slit of the spectrometer. Without changing $a$ and $a'$, the crystal is moved in the range 0–25 mm, so that different propagation planes of PDC are always imaged to the input slit of the spectrometer. The bandwidth filter (BWF) deflects most of the pumping beam power. Adapted from Haderka, O., Machulka, R., Perina, J., Allevi, A., Bondani, M., 2015. Spatial and spectral coherence in propagating high-intensity twin beams. Sci. Rep. 5. http://dx.doi.org/10.1038/srep14365.

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712.5 nm has been registered by means of an iCCD camera (Andor iStar DH334T, 13 × 13 μm² pixel size). The spectral resolution of the system was 0.03 nm/pixel. The spectrum was centered approximately around the PDC degenerate wavelength (698 nm). A 50 μm-wide slit was used throughout the experiment. The camera was operated at the maximum gain and maximum A/D conversion speed (5 MHz). The gating window of the camera was set to 5 ns, synchronous with the laser pulses, to ensure the detection of single shots of the PDC in each image. At each z-axis setting, a sequence of 10000 camera frames (with full-resolution) was taken. We performed a scan of the spatial and spectral correlations as functions of the z-axis position. For each z-axis position, 100 points in each image were taken and processed in the same way explained above to calculate the intensity autocorrelation and cross-correlation functions.

In panels (A)–(C) of Fig. 12 we show the spatial positions of the cross-correlation peaks against the spatial positions of the corresponding autocorrelation peaks. Note that, to take into account the different divergences of the PDC beam at the transition from the near-field to the far-field configurations, in each panel the real positions on the camera were normalized to the size (FWHM) of the corresponding PDC beam. From the figure, we note a sharp diagonal around the position z = 0 mm, which corresponds to the near field. The location of the near field at this position is also confirmed by the direct observation of the crystal output face at the plane of the input slit of the monochromator when the monochromator is switched to the imaging mode. Moving the crystal away from the monochromator leads to a blurring of the diagonal. Around position z = 4 mm, the diagonal is completely lost, so there is no spatial correlation in the positions of the peaks. Moving the crystal even further to higher z values, an antidiagonal character of the correlations (anticorrelation) is gradually established. Toward position z = 24 mm, the antidiagonal becomes sharper, thus indicating the approach to the far-field momentum correlations. We are not exactly in the far-field configuration reached at z = ∞, but the far-field character of the correlations is clearly established at a distance exceeding 1 cm from the crystal. Note that in the transient area (between z = 4 mm and z = 9 mm), the cross-correlations in the PDC field are nearly lost only in space. On the contrary, spectral cross-correlations remain unchanged as documented in panels (D)–(F) of Fig. 12. Here the tight anticorrelations emerging from the energy conservation are preserved at all z positions.

In addition to the position of the peaks and in analogy with the investigation at different pump powers, Fig. 13A reports the spatial FWHM
Fig. 12  Spatial (panels A–C) and spectral (panels D–F) positions of the autocorrelation peak vs the corresponding cross-correlation peak for different z-axis positions: (A) and (D), $z = 0$ mm (near field); (B) and (E), $z = 5$ mm; (C) and (F), $z = 24$ mm (far field). (A)–(C) Normalized to the size of the corresponding PDC beam FWHM width to scale them to the divergence of the beam. (D)–(F) The whole registered bandwidth (683.4–712.5 nm) in both coordinates. Adapted from Haderka, O., Machulka, R., Perina, J., Allevi, A., Bondani, M., 2015. Spatial and spectral coherence in propagating high-intensity twin beams. Sci. Rep. 5. http://dx.doi.org/10.1038/srep14365.
 widths of the autocorrelation and cross-correlation areas as functions of the $z$-axis position. We note that the near-field configuration occurs at the left edge of the plots. In the near-field configuration, the autocorrelation and cross-correlation functions have nearly identical spatial widths. Moving toward the far field ($z = 24$ mm) the spatial autocorrelation width exhibits a mild growth, while the spatial cross-correlation width grows rapidly up to the PDC beam size, which was determined from the spatial extent of the beam as obtained from a large number of accumulated images at each $z$-position. As expected, the increase of the spatial cross-correlation width happens at the distances at which we find the completely blurred spatial diagonals (see for instance Fig. 12B). Going further toward the far field, momentum anticorrelations get established resulting in a gradual decrease of the spatial cross-correlation width. At variance with space, the autocorrelation and cross-correlation spectral FWHM widths shown in Fig. 13B do not exhibit any kind of evolution at different $z$-positions.

The behavior of the spatial cross-correlation widths can be also used to investigate the Fedorov ratio plotted in Fig. 14. As presented previously, at macroscopic level this ratio is again defined as the ratio of the width of the whole PDC beam to the cross-correlation width. While the spatial Fedorov ratio gives the number of paired degrees of freedom in the near field and far field, the blurring of the intensity cross-correlations prevents the determination of this number in the transition region. Nevertheless, it can be used to quantify the strength of blurring. We remark that the evolution of the spatial
Fedorov ratio is very similar to that reported by Just et al. (2013) in the single-photon regime.

3.5 Measurement of Nonclassical Correlations

As already mentioned in Section 2, the nonclassical character of TWB states lies in the existence of strong correlations between several degrees of freedom, such as linear position and momentum, number of photons, frequency, polarization, angular momentum, etc. Among them, photon-number correlations have been the subject of intensive investigations for state characterization. In many cases, the detection was performed by means of single or arrays of avalanche photodiodes so that the possibility to recover the correlation of the number of photons was quite straightforward (Avenhaus et al., 2010; Ivanova et al., 2006), although the intensity range actually investigated by these systems was limited to much less than one mean photon (Kalashnikov et al., 2011). Also the generation and characterization of more intense TWBs have received a lot of attention, especially in view of possible applications to quantum technology. Indeed, pulsed optical states endowed with sizeable numbers of photons represent a useful resource in the field of quantum information, as they are robust with respect to losses.

Among the nonclassicality criteria for TWB states that can be expressed in terms of photon numbers, the existence of subshot-noise photon-number correlations is routinely investigated. As already explained in Section 2, this criterion is based on the calculation of the noise reduction factor, whose values attain lower-than-1 numbers in the case of nonclassical correlations.
Indeed, subshot-noise photon-number correlations have been revealed in macroscopic TWBs by means of different kinds of detectors, such as pin photodiodes, CCD cameras, but also amplified pin-photodiodes in a lower intensity regime (more than $10^3$ detected photons). However, it is worth noting that the use of these macroscopic detectors to test nonclassicality is fragile, as a reliable a priori calibration is required and the contribution of electronic noise is not negligible and must be subtracted from the signal. On the contrary, photon-number resolving detectors, such as HPDs, can be easily calibrated by means of self-consistent methods and are endowed with a lower level of noise, which in many cases can be neglected thus making the evaluation of the nonclassicality criteria straightforward and definitely more reliable. The main drawback given by this kind of detectors is that in general their quantum efficiency is lower than 50% and thus reconstruction algorithms to describe the nonclassicality of TWBs cannot be applied. However, as already anticipated in Section 2, it is possible to express the same quantities defined for incident photons, such as correlation coefficient and noise reduction factor, in terms of detected photons. This makes the analysis of the experimental data more straightforward and less time demanding. In our research activity, we adopted such a strategy several times to characterize the nonclassicality of multimode TWB states generated in the mesoscopic domain. In particular, here we present three different nonclassicality criteria based on detected-photon-number correlations, namely the Schwarz inequality in Eq. (16), the noise reduction factor in Eq. (17) and the high-order correlation inequality in Eq. (18), and we discuss the conditions suitable for their application.

### 3.5.1 Experimental Setup

The experimental setup is sketched in Fig. 15. The fourth harmonics (at 262.2 nm) of the already described Nd:YLF laser was sent to a 6-mm-long BBO crystal (cut angle = 46.7 degree) to produce PDC in a quasi-collinear interaction geometry. Two twin portions around frequency degeneracy (i.e., at 523 nm) were spatially and spectrally filtered by a variable iris and a bandpass filter, respectively. In both arms the light was then focused into a 600-μm-core multimode fiber by means of an achromatic doublet and delivered to a PNR detector. As the PNR detectors we used two HPDs, whose operational mechanism has been already described in Appendix. We acquired sequences of 100,000 shots at different mean values of the fourth harmonics, whose energy was changed by means of a half-wave plate followed by a polarizing cube BS.
3.5.2 Results

In Section 3.3, we have shown how our detection apparatus determines the number of photons at each laser shot correctly enough to properly reconstruct the statistics of detected photons. The possibility to have access to the number of photons is fundamental to reveal shot-by-shot photon-number correlations between the parties. To this aim, in Fig. 16 we show the experimental data (colored dots) obtained by evaluating the high-order correlation functions up to the fourth order in the symmetrized form $g_{hk}^{(s)} = 1/2(g_{hk} + g_{kh})$. The same figure reports the theoretical expectations (colored open symbols) calculated according to Eqs. (15) by using the experimental values of the mean number of detected photons, of the number of modes $\mu$.

Fig. 15 Sketch of the experimental setup for the measurement of the nonclassical correlations.

Fig. 16 High-order correlations as functions of the mean number of photons per arm. (A) $g^{11}$ (black symbols) and $g^{21}$ (red symbols); (B) $g^{22}$ (blue symbols) and $g^{31}$ (magenta symbols). Experimental data are plotted as dots, whereas theoretical expectations are shown as empty circles + line.

3.5.2 Results

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and of the overall detection efficiency $\eta$. In particular, the mean number of photons has been directly determined by means of the self-consistent method of analysis (see Appendix). $\mu$ has been obtained as

$$\mu = \frac{\langle m \rangle}{\sigma^2(m) - \langle m \rangle}$$

(42)

from the first two moments of statistics. Finally $\eta$ has been calculated by the experimental value of the noise reduction factor, that is, by imposing

$$R = \frac{\sigma^2(m_1 - m_2)}{\langle m_1 \rangle + \langle m_2 \rangle} = 1 - \eta.$$ 

(43)

Notice that in both panels of Fig. 16, which correspond to the different correlation orders, the experimental data exhibit a very good agreement with theory, thus testifying the usefulness of high-order correlations for state characterization.

However, photon-number correlations are not sufficient markers of nonclassicality. As anticipated above, other quantities based on the number of photons and satisfying boundary conditions are usually considered. In addition to the already existing criteria, Allevi et al. (2012) have introduced the $B$ parameter inequality of Eq. (18), based on high-order photon-number correlations for detected photons and thus only in terms of measurable quantities.

Fig. 17 reports the noise reduction factor, the Schwarz inequality and the high-order inequality all together, in order to get a fair comparison between them. All the inequalities are calculated for detected photons and are plotted as functions of the mean number of photons per arm. The experimental data are shown as dots, whereas the theoretical predictions are plotted as empty circles + line. Notice the good agreement between data and theory for all the considered criteria. The worse superposition in the case of the $B$-parameter high-order inequality arises because such a criterion involves high-order correlations, and it is thus much more sensitive to noise in the experimental system. However, it is remarkable that the values of the high-order correlation criterion are reasonably above 1, that is, the classical boundary, thus testifying the nonclassical character of TWB. In particular, we notice that such $B$-parameter criterion well emphasizes the presence of nonclassicality for high mean photon numbers, at variance with Schwarz inequality $S$ that better discriminates at low photon numbers. It is worth also noting that in
due to their nonclassical character, TWB states represent a useful resource in quantum state engineering for applications to quantum information and metrology, both at the single-photon level and in the mesoscopic and macroscopic domains. For instance, it is well known that such states can be used to produce nonclassical states by means of conditional measurements exploiting the existence of correlations between signal and idler. In particular, when a given number of photons is detected in the idler, the state of the signal is irreversibly modified. As the original state (the TWB) is quantum correlated, the generated conditional states can show sub-Poissonian statistics, which is nonclassical. This feature has been experimentally demonstrated in different intensity regimes, but mostly at single-photon level. However, it is worth noting that the use of photon-counting detectors instead of single-photon detectors offers the possibility not only to enhance the heralding of single-photon states by suppressing higher photon-number components (Christ and Silberhorn, 2012; Haderka et al., 2004;
Peřina et al., 2001), but also to perform multiple photon-counting operations in order to produce quasi-Fock states endowed with a number of photons larger than 1 (Peřina et al., 2013).

The extension of the experimental results presented so far to a more mesoscopic photon-number domain, where the states are more robust with respect to losses, is thus desirable and still subject to active research. Moreover, we notice that, since the conditioning operation is non-Gaussian (Allevi et al., 2010) the resulting conditional states are non-Gaussian, thus acquiring an additional interesting feature for quantum information applications. Here we show that conditional states endowed with both sub-Poissonian statistics and non-Gaussian character can be generated by exploiting the TWB states already described in Section 3.5. We performed conditional measurements by selecting 0, 1, 2, or 3 photons in the idler beam, thus obtaining the conditional states $\rho_{m_2=0}$, $\rho_{m_2=1}$, $\rho_{m_2=2}$, and $\rho_{m_2=3}$ in the signal beam, respectively (Peřina et al., 2013). This choice gives us the possibility to show that our detection apparatus can be used to perform conclusive photon subtractions by exploiting the photon-counting capability of HPDs. Among the conditioning values, $\rho_{m_2=0}$ represents a sort of reference level. In fact, in this case the conditional states remain super-Poissonian and Gaussian. Indeed such an operation is Gaussian, at variance with all the other conclusive operations.

We also notice that the results obtained in the case $\rho_{m_2=1}$ are definitely different from those that can be achieved by employing single-photon detectors operated in Geiger ON/OFF mode, as we do not need to assume that the output states contain 1 photon at most. Moreover, the conditioning operations presented here are useful to investigate in which way the nonclassical nature of the conditional states depends on the different experimental parameters involved in their production. In order to quantify the nonclassicality of the conditional states, here we consider their Fano factor, which is defined as $F_n = \sigma^2_n/\langle n \rangle$. Actually, in the following we will consider the analogous quantity written in terms of detected photons. Indeed, it is possible to demonstrate that $F_m$ can be easily linked to the corresponding quantity for incident photons, namely $F_m = \eta F_n + (1 - \eta)$. Both quantities are equal to 1 in the case of Poissonian light. If $F_n < 1$, the light is nonclassical and is called sub-Poisonian light. Note that the value 1 for the boundary between classical and nonclassical behaviors still holds for detected photons. It is interesting to notice that the minimum value of this expression coincides with the minimum value of the noise reduction factor $R = 1 - \eta$. In Fig. 18 we plot
the Fano factor of the conditional states as a function of the mean number of photons detected in the signal arm before conditioning. Different colors correspond to different conditioning values. In particular, it is worth noting that the Fano factor attains its lowest values either at increasing conditioning values or at decreasing the mean values of the unconditioned states. In general, the agreement between the experimental data and the theoretical expectations is good. For some points the discrepancies can be ascribed to possible fluctuations due to the low number of experimental data giving the conditional states.

Besides being sub-Poissonian, and hence nonclassical, the conditional states generated from TWB are also non-Gaussian, which means that they are not described by a Gaussian Wigner function. There are different ways to quantify the non-Gaussianity of a state. Here we calculate the $\delta_\rho$ relative entropy of non-Gaussianity introduced by Genoni et al. (2008) in the case of detected photons. For a generic state, this quantity is defined as the difference between the von Neumann entropy $S[\sigma] = -Tr[\sigma \ln(\sigma)]$ of a reference Gaussian state $\sigma$ and that of the state under investigation, namely

$$\delta_\rho = S[\sigma] - S[\rho].$$

Note that the Gaussian reference state is chosen to have the same mean value and covariance matrix as the state under investigation. According to this
definition, the estimation of the non-Gaussianity implies the complete knowledge of the density matrix of the state. However, there are some cases in which the calculation is simpler, such as for diagonal states, whose von Neumann entropy (and hence non-Gaussianity) depends only on the photon-number statistics. In the case of TWB, which is in principle a diagonal state, this simplification does not hold because of its multimode nature. Thus, to calculate the non-Gaussianity amount the only possibility is to evaluate the expected density matrix of the states on the basis of the experimental values. In particular, in this calculation the mean number of photons of the conditional states, the number of modes and the quantum efficiency are involved.

Fig. 19 reports the non-Gaussianity of the conditional states as a function of the mean number of photons detected in the signal arm before conditioning. The dots represent the values of non-Gaussianity obtained by using the experimental mean values of the conditional states, whereas full circles + line have been obtained by calculating the expected mean values on the basis of the other experimental parameters, namely the mean number of photons of the unconditioned states, the number of modes, and the quantum efficiency. The agreement between the two sets of points is really good, thus remarking the reliability of the performed conditioning operations. Moreover, by comparing the behavior of non-Gaussianity shown in Fig. 19 to that

![Fig. 19](image_url)

**Fig. 19** Non-Gaussianity of the conditional states as a function of the mean number of photons detected in the signal arm before conditioning. Different colors correspond to different conditioning values. Black: $m_2 = 0$, red: $m_2 = 1$, blue: $m_2 = 2$ and green: $m_2 = 3$. Experimental data are shown as dots, whereas theoretical expectations as empty circles + line.
of the Fano factor plotted in Fig. 18, it is quite clear that their evolutions are complementary. In fact, the higher values of non-Gaussianity appear for the smaller values of Fano factor.

### 4.2 Ghost Imaging

Ghost imaging is a technique based on the correlated nature of both classical and quantum fields, which allows the retrieval of the image of an object not interacting with the measured light (for a review see Gatti et al., 2008). The ghost-imaging protocols rely on the properties of correlation in the propagation direction and in the intensity between two spatially multimode fields (D’Angelo and Shih, 2005; Gatti et al., 2006, 2007; Pittman et al., 1995; Shih, 2007). The technique was originally introduced by Klyshko (1988) with the idea of exploiting quantum entanglement in signal-idler photon pairs generated by PDC. According to the original scheme, the photons of a pair are sent to two different imaging systems: the signal illuminates an object and then is detected by a “bucket” detector, whereas the idler is revealed by a position-sensitive detector. Information about the object is obtained from the coincidences of signal-idler photon pairs as functions of the transverse position of detector in the idler-arm (Abouraddy et al., 2001). By changing the optical elements in the two arms, it is possible to reconstruct either the image of the object (ghost image) or its diffraction pattern (ghost diffraction) (Brambilla et al., 2004; Ferri et al., 2005). These results were theoretically extended to the macroscopic realm, where a large number of photon pairs form entangled beams and the information about the object is retrieved by the correlation of the intensity fluctuations (Gatti et al., 2003). From the experimental point of view, the correlated light required for ghost imaging has been generated by several physical systems, starting from PDC in single-photon regime (D’Angelo and Shih, 2005; Pittman et al., 1995), passing through chaotically seeded down-conversion (Bondani et al., 2008; Puddu et al., 2007), and ending with macroscopic thermal beams (Ferri et al., 2005; Valencia et al., 2005; Zhang et al., 2005). With a suitable choice of the optical setup, all these systems lead to the production of a ghost image. Of course, according to the chosen conditions, the processes perform differently in terms of visibility, resolution, and signal-to-noise ratio (Erkmen and Shapiro, 2009, 2010; Ferri et al., 2005; O’Sullivan et al., 2010).

Bondani et al. (2012) presented the first experimental implementation of the ghost-imaging scheme obtained by means of an intense TWB, as
proposed by Gatti et al. (2003), both by means of intensity correlations (as usual) and of high-order correlations. That paper demonstrated that, in analogy with classically correlated multimode light, the visibility of the ghost image increases at increasing the order of correlation, whereas the signal-to-noise ratio is expected to have a maximum at a certain value of the order of correlation (Chan et al., 2009; Chen et al., 2010; Iskhakov et al., 2011). The obtained results are here summarized.

### 4.2.1 Experimental Setup

Let’s consider a ghost-imaging scheme in which a 2D-object is located in the plane $x_o$ in the signal-arm at a distance $d_1$ from the crystal and an imaging lens is located in the idler-arm at a distance $d_2$ from the crystal and $d_3$ from the detector (see Fig. 20). The light transmitted by the object in the signal-arm is integrated by a “bucket” detector in the plane $x_{si}$, while that propagating in the idler-arm is detected by a position-sensitive detector in the plane $x_{id}$. As the sensor Bondani et al. (2012) used a 16-bit EMCCD (ImagEM, Hamamatsu, Japan) synchronized to the laser, to measure both arms of the setup. We decided to calculate the “bucket” values by integrating the light passing through the object and imaged on the sensor by lens $L_3$ in Fig. 20 with a magnification factor $M = 0.47$. This strategy allowed the authors to have a cross-check for the ghost image with a conventional image. As the object we used a mask containing four small holes of about 450 μm diameter separated by about 450 μm from each other. We chose the distances $d_1 = 30\, \text{cm}$, $d_2 = 30\, \text{cm}$, and $d_3 = 60\, \text{cm}$ in order to satisfy the thin

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**Fig. 20** Scheme of the experimental setup for the ghost-image investigation. $BBO$, nonlinear crystal; $IF$, interference filter; $M$, mirror; $L_j$, lens; $t(x)$, transmissivity of the object. Adapted from Bondani, M., Allevi, A., Andreoni, A., 2012. Ghost imaging by intense multimode twin beam. Eur. Phys. J. Spec. Top. 203 (1), 151–161. ISSN 1951-6401. [http://dx.doi.org/10.1140/epjst/e2012-01540-4](http://dx.doi.org/10.1140/epjst/e2012-01540-4).
lens equation \( M = d_3/(d_1 + d_2) \) for the \( M \) magnification factor leading to \( M = 1 \). Signal and idler beams were then detected by two distinct regions of the EMCCD.

4.2.2 Images

The reconstruction of the ghost image is achieved by calculating the normalized intensity correlation function,

\[
g^{(2)}(x_{id}) = \frac{\int d x_{si} \langle \hat{I}_{id}(x_{id}) - \langle \hat{I}_{id}(x_{id}) \rangle \rangle \langle \hat{I}_{si}(x_{si}) - \langle \hat{I}_{si}(x_{si}) \rangle \rangle}{\int d x_{si} \langle \hat{I}_{si}(x_{si}) \rangle \int d x_{id} \langle \hat{I}_{id}(x_{id}) \rangle},
\]

(45)

where \( \hat{I}_{i}(x_{i}) = \hat{c}_{i}^{\dagger}(x_{i}) \hat{c}_{i}(x_{i}) \) \( (i = si, id) \) is the intensity operator of the \( i \)-th beam at the detection plane. The field operators at the detection planes are connected to those \( \hat{b}_{i}(x) \) at the output face of the crystal by the propagators \( h_{id}(x_{id}, x_{id}') \) and \( h_{si}(x_{si}, x_{si}') \) describing the optical setup (Goodman, 1996)

\[
\hat{c}_{i}(x_{i}) = \int d x_{i}' h_{i}(x_{i}, x_{i}') \hat{b}_{i}(x_{i}').
\]

(46)

The explicit expression of the propagators for the setup in Fig. 20 is calculated in Bondani et al. (2008). Introducing \( 1/f = 1/(d_1 + d_2) + 1/d_3 \), the final expression of the correlation function becomes

\[
g^{(2)}(x_{id}) \propto |t(-x_{id})|^2 \int d x_{si} n_{PDC} \left( \frac{2 \pi x_{si}}{\lambda f} \right) \left[ 1 + n_{PDC} \left( \frac{2 \pi x_{si}}{\lambda f} \right) \right]
\]

(47)

In order to reconstruct the ghost image by computing the normalized second-order correlation function in Eq. (47), we recorded many EMCCD frames, each one containing both the single-shot map in the idler-arm and the corresponding bucket value in the signal-arm.

In Fig. 21A we show the image of the object obtained by the EMCCD on the signal-side and in Fig. 21B the ghost image recovered by cross-correlation. What we can see is that the object size is almost correctly reproduced, although the resolution is not very high due to the speckle size that is comparable to the object size. To explore the effects of the size and shape of the coherence areas on the image reconstruction, we can calculate the correlation of the bucket with the portion of the EMCCD sensor containing the image of the object. The result is displayed in Fig. 21C: the image obtained from autocorrelation is very similar to the ghost image, due to the
fact that the correlation is sensitive to the speckle structure of the field and hence the resulting image can be viewed as the convolution between the hole size and the speckle size.

In order to quantify the quality of the imaging protocol, in the following we consider the visibility (VIS) and the signal-to-noise ratio (SNR). The former quantity is defined as

\[
\text{VIS} = \frac{g^{(2)}_{\text{IN}} - g^{(2)}_{\text{OUT}}}{g^{(2)}_{\text{IN}} + g^{(2)}_{\text{OUT}}},
\]  

(48)

where \(g^{(2)}_{\text{IN}}\) is the mean value of \(g^{(2)}\) inside the image and \(g^{(2)}_{\text{OUT}}\) is the mean value in a portion of the matrix that does not contain the image. The latter quantity is defined as the “contrast” of the image divided by its noise, namely

\[
\text{SNR} = \frac{g^{(2)}_{\text{IN}} - g^{(2)}_{\text{OUT}}}{\sqrt{\text{Var}(g^{(2)}_{\text{IN}} - g^{(2)}_{\text{OUT}})}},
\]  

(49)

We note that in the case presented above, the visibility of the ghost image is actually very low, due to the high total number of modes (temporal and spatial) involved in the calculation of the correlations. In fact, we expect that VIS in Eq. (48) scales as \(1/\mu_{\text{tot}}\) (Gatti et al., 2004; Lopaeva and Chekhova, 2010). A way to increase the visibility is to calculate ghost images with high-order correlations, as we will see in the next section.
4.2.3 High-Order Ghost Imaging

Recently, there has been an increasing interest in the evaluation of high-order correlation functions applied to the calculation of ghost images. In the case of classically correlated multimode pseudo-thermal light it has been demonstrated that the visibility of the ghost image increases at increasing the order of correlations, while the signal-to-noise ratio is expected to have a maximum at a certain value of the order of correlation (Chan et al., 2009; Chen et al., 2010; Iskhakov et al., 2011).

The same kind of considerations can be applied also to our multimode TWB that represents the case of intense nonclassically correlated light. By calculating high-order correlations for TWBs, Brida et al. (2011) and Allevi et al. (2012) have observed that those correlations depend on the mean number of photons in the states. Nevertheless, the dependence disappears at high mean photon numbers and the correlation functions reduce to those of multimode thermal light (Allevi et al., 2012). In particular, in the limit of high numbers of photons we can write:

\[
g^{n,m} = \frac{\langle \tilde{I}_{st} \cdot \tilde{I}_{id} \rangle^n \langle \tilde{I}_{si} \cdot \tilde{I}_{id} \rangle^m}{\langle \tilde{I}_{st} \rangle^n \langle \tilde{I}_{id} \rangle^m} \approx \prod_{k=1}^{n+m-1} \left(1 + \frac{k}{\mu}\right),
\]

where we have assumed \(\mu\) equally populated modes in the field. By following the results for classical light, from Eq. (50) we expect that also for the multimode TWBs the VIS calculated for high-order correlations increases at increasing the order of correlations and that the corresponding SNR has a maximum at a certain value of the order of correlation. Fig. 22 reports the
values of VIS (panel A) and SNR (panel B) calculated from our experimen-
tal data. The experimental behavior qualitatively fits the theoretical expec-
tations, even if the SNR is rather noisy. This problem could be overcome by increasing the number of images employed for correlations. Note that, since in the intensity regime of our experiment the calculated high-order ghost images have the same VIS and SNR as the classically correlated light, to have a real improvement in the protocols by the use of nonclassical light, one should employ a different approach, such as the subshot-noise ghost imaging introduced by Brida et al. (2010) or the ghost imaging based on the variance of the difference of detected photons introduced by Lopaeva and Chekhova (2010) and Brida et al. (2011). To this aim, an exact calibration of the camera is required to obtain a reliable estimation of the subshot-noise reduction factor, as already performed in Jedrkiewicz et al. (2004) and Brida et al. (2009a).

5. CONCLUSION

In this review, we presented the investigations performed on optical TWBs over the last 10 years in the laboratory of Quantum Optics at the University of Insubria. Such states were generated and characterized both in the mesoscopic and macroscopic intensity regimes, also including pump depletion. The achievement of the high-intensity levels was made possible by the availability of a pulsed laser source capable of producing few-ps UV pulses with high-peak intensity values (up to some GW/cm²). Thanks to this source, we were able to produce bright TWBs, whose coherence properties have been easily characterized by means of an imaging spectrometer and two different kinds of amplified cameras. Moreover, thanks to our long-lasting experience with the use of photon-number resolving detectors, the statistical and nonclassical features of mesoscopic TWBs were extensively investigated. The results presented in the chapter are not limited to the characterization of TWB states, but also testify their possible application to the fields of quantum state engineering, quantum information, and quantum imaging. In particular, we showed that mesoscopic multimode TWBs are useful to produce sub-Poissonian and non-Gaussian states by means of conditional operations and, finally, we described the first implementation of a ghost-imaging protocol based on macroscopic TWBs.
A.1 Detection Distributions

For the photon-number-resolving detectors described above we can assume that photodetection is performed with quantum efficiency $\eta$ and no dark counts. The probability operator-valued measure (POVM) of each detector, describing the number of detected photons $m$ as a function of the number of incident photons $n$ is thus given by a Bernoullian convolution of the ideal number operator spectral measure $\hat{P}_n = |n\rangle \langle n|$

$$\hat{\Pi}_{m_j} = \eta_j^{n_j} \sum_{n_j=m_j}^{\infty} (1 - \eta_j)^{n_j - n_j} \left( \begin{array}{c} n_j \\ m_j \end{array} \right) \hat{P}_{n_j}, \quad (A.1)$$

with $j = 1, 2$. The moments of the distribution are evaluated by means of the operators

$$\hat{m}_j^p = \sum_{m_j} m_j^p \hat{\Pi}_{m_j} = \sum_{n_j=0}^{\infty} (1 - \eta_j)^n G_{\eta_j}(n) \hat{P}_{n_j}, \quad (A.2)$$

where

$$G_{\eta}(n) = \sum_{m=0}^{n} \left( \begin{array}{c} n \\ m \end{array} \right) \left( \begin{array}{c} \eta \\ 1 - \eta \end{array} \right)^m m^p. \quad (A.3)$$

Of course, since they are operatorial moments of a POVM, we have, in general, $\hat{m}_j^p \neq \hat{m}_j^p$. The first two moments correspond to the operators

$$\hat{m}_j = \eta_j \hat{n}_j$$
$$\hat{m}_j^2 = \eta_j^2 \hat{n}_j^2 + \eta_j(1 - \eta_j) \hat{n}_j. \quad (A.4)$$

Thus for the variance of the detected statistics we have

$$\sigma^2(m_j) \equiv \langle \hat{m}_j^2 \rangle - \langle \hat{m}_j \rangle^2 = \eta_j^2 \sigma^2(n_j) + \eta_j(1 - \eta_j) \langle \hat{n}_j \rangle. \quad (A.5)$$

As presented in Mandel and Wolf (1995) and Agliati et al. (2005), the resulting statistics of the number of electrons generated by the primary photon interaction (detected photons) that occurs in the detector, $p_m$, is linked to that of the number of photons in the light under measurement, $p_n$, by
In principle, \( p_m \neq p_n \), that is the functional form of the probability distribution of the detected photons is not necessarily the same as that of the photons (Casini and Martinelli, 2013). Indeed, it is not obvious that a Poissonian (or a thermal) distribution for the number of photons originates a Poissonian (or a thermal) distribution of detected photons. As a matter of fact, for all the fields described by a classical photon-number distribution (including signal and idler in the TWB) the statistics is invariant under the convolution in Eq. (A.6). This is definitely not true for purely quantum states, such as Fock states.

### A.2 Amplification Distributions

For almost any detector, after the primary detection process the number of generated photoelectrons undergoes amplification. To guarantee the linearity of the response, both the amplification internal to the detector (gain) and that of the electronics processing its output must be independent of the number of detected photons. If we can assume that the spread of the single photoelectron peak in the final electronic output is negligible as compared to its mean value, the relation linking the statistics of the photoelectrons to that of the final voltage outputs, \( \nu \), derived by Andreoni and Bondani (2009) is

\[
p_v = \frac{1}{\gamma} p_{\gamma m},
\]

being \( \gamma \) a conversion coefficient describing the overall \( m \)-to-\( \nu \) conversion process. The first two moments of the distribution in Eq. (A.7) reported by Agliati et al. (2005) are given by

\[
\langle \nu \rangle = \gamma \langle m \rangle; \quad \sigma^{(2)}_\nu = \gamma^2 \sigma^{(2)}_m,
\]

where the symbols are defined as in Eq. (A.5).

To obtain the value of \( \gamma \) we calculate the Fano factor for the output voltages, defined as \( \tilde{F}_\nu = \sigma^{(2)}_\nu / \langle \nu \rangle \). By using Eq. (A.5) and (A.8) we find

\[
\tilde{F}_\nu = \frac{\gamma^2 [\eta^2 \sigma^{(2)}_n + \eta(1-\eta)\langle n \rangle]}{\gamma \eta \langle n \rangle} = \gamma \eta \tilde{F}_n + \gamma (1-\eta) = \gamma \eta Q + \gamma,
\]
where we have rewritten the Fano factor for the photons, \( F_n = \sigma^{(2)}(n)/\langle n \rangle \), as \( \tilde{F}_n = 1 + Q \) in terms of the Mandel Q-factor. By multiplying and dividing Eq. (A.9) by \( \langle n \rangle \) and reusing the above equations we get

\[
\tilde{F}_v = \frac{Q}{\langle n \rangle} \langle \nu \rangle + \gamma, \tag{A.10}
\]

in which \( Q/\langle n \rangle = (\sigma^{(2)}_n - \langle n \rangle^2)/\langle n \rangle^2 \) only depends on the specific state under measurement. Thus Eq. (A.10) shows a linear dependence of the Fano factor \( \tilde{F}_v \) on \( \langle \nu \rangle \). Note that the proportionality coefficient is zero for Poissonian light, positive for classical super-Poissonian light and negative for non-classical sub-Poissonian light. This linearity can be verified by repeatedly measuring a field upon inserting filters with different transmittance in front of the detector. In fact the insertion of filters modifies \( \langle \nu \rangle \), leaving \( Q/\langle n \rangle \) unchanged. We can thus obtain \( \gamma \) from the fit of the experimental data. Once \( \gamma \) is evaluated, it is possible to find the photoelectron distribution by dividing the \( \nu \) output values by the experimental value of \( \gamma \) and re-binning the data in unitary bins. The procedure has the advantage of being self-consistent as the value of \( \gamma \) is obtained from the measurements on the same field under investigation (Andreoni and Bondani, 2009; Bondani et al., 2009a) and does not require an independent calibration.

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**REFERENCES**


INDEX

Note: Page numbers followed by “f” indicate figures, and “t” indicate tables.

A
Absorption imaging techniques, 390
Acetylene, dissociative electron attachment, 607–609
Acousto-optic modulators (AOMs), 18
ac-Stark shift, 475
Active stabilization, 15–17, 16f, 32–34
Adams–Bashforth–Moulton multistep algorithm, 397
Adiabaticity, 185–187
Adiabatic potentials for rf-dressed atoms, 203–212
Ioffe–Pritchard trap, 212–218, 216f
isomagnetic surface, role of, 208–209, 209–210f
loading from magnetic trap, 209–212, 211f
local coupling, 205–208
time-averaged, 226–233, 227f
Adiabatic rapid passage (ARP), 189–190
Adiabatic state, 204
Adiabatic theorem, 204
Adiabatic traps, applications of, 189–190
Affine approach, 383–384, 420–422
to numerical simulations, application, 394–405
Bures distance, 395–396, 403–405
in rotating trap, 396–403
time-dependent TF approximation accuracy, 394–405
time evolution of, 394
Affinely transformed wave function, GP equation, 383–384
Airy wave packet, 422
Alkali metal atomic vapors, 19
Andor iStar DH334T, iCCD camera, 81–82
Andor Shamrock 303i, imaging spectrometer, 81–82
Angular momentum, 190–194
of BEC, 414–415
conservation, 408–409
matrix, 409
rotation of spin operators, 192–193, 193f
rotation operators, 191–192
spin operators, 190–191
time-dependent rotations, 193–194
AOMs. See Acousto-optic modulators (AOMs)
Argon buffer gas, 19
ARP. See Adiabatic rapid passage (ARP)
Arrhenius law, 575–578, 576f, 578f, 591–592
Atomic collisions, 511, 515–516
Atomic mirrors, 476
Atomic vapor, multidimensional coherent spectroscopy of
one-and zero-quantum 2D spectra, 19–22
three-dimensional coherent spectroscopy, 26–29
two-quantum 2D spectra, 22–26
Atom–light coupling, 441
Atoms–optical coupling, 441
atom–light nanofiber interactions, 464–469
atomic dipole trap, 469–472
atom–surface interactions, 465–467
dispersive measurements, 473–474
ground state coherence, 476
higher-order modes, trapping, 479
linear and nonlinear spectroscopy, 467–469
possible heating mechanisms, 479–480
reflectivity, 476–478
state-sensitive and state-insensitive traps, 475–476
transmission spectroscopy, 472–473
trapping, 469–480
Autoionization, 580
Avalanche photodetectors, 50
B
Bandwidth filter (BWF), 81–82
BBO crystal. See β-barium borate (BBO) crystal
BDE. See Bond dissociation energy (BDE)
Beam-splitter (BS) Hamiltonian, 277–278, 278f
Beat frequencies, 18
BEC. See Bose–Einstein condensate (BEC)
derivation by nonlinear filtering, 351–354
characteristic function technique, 352–354
conditional expectation value, 351–352
quantum optical derivation, 346–351, 346f, 349f
Bernoullian process, 56, 69–70
Bessel functions, 449–450, 453
β-barium borate (BBO) crystal, 68–69, 73–74
Biexciton binding energy, 38–39
Biexciton states, 35–36
Bloch equations, 34
Bloch–Siegert shift, 253–254
Bohr radius, 442–443
Bond dissociation energy (BDE), 596–597
Bose–Einstein condensate (BEC), 209, 218, 376–377
angular momentum of, 414–415
energy of, 412–413
GP equation, affine transformation
affinely transformed wave function, 383–384
center-of-mass motion elimination, 380–381
classical trajectory interpretation, 385–386
linear transformation, scaling approach, 381–383
nonrotating trap, 384
settings, 379–380
in harmonic trap, 411–412
macroscopic wave function, in
time-dependent TF approximation, 386–393
for dynamical evolution, 388–390
ellipsoidal density distribution
boundaries of, 389f
for initial ground state, 386–387
integrated density distributions for
time-of-flight pictures, 390–391
internal dynamics of, 392
rotating and vortex-free condensates,
392–393
time evolution of, 378–393
Bragg resonators, 487–488
Bragg scattering, 477–478
Breit–Wigner formula, 557
Bromine (Br₂), dissociative electron attachment, 592
Bromouracil, 554–555
5-Bromouridine, 554–555
Brownian motion damping, 281
Bulk GaAs, 30
Bures distance, affine approach, 395–396, 401f
on setup parameters, dependence of, 403–405
time dependence of, 398f, 401
Burning a hole, 7
BWF. See Bandwidth filter (BWF)

C
Caldeira–Laggett approach, 271
Canonical form, of matrix differential equation, 406
Carbon monoxide (CO), dissociative electron attachment, 582–584, 584f
Carbon tetrafluoride, dissociative electron attachment, 595–597, 595–596f
Casimir–Polder (CP) potentials, 465
Cavity–optomechanical systems, 269–290
open system dynamics, 279–286
master equation and stochastic master equation, 281–283
phase space description, 283–286, 285f
quantum Langevin equations, 280–281
optomechanical phase diagram, 286–290, 287f
physical system, 269–275, 269f
cavity-optomechanical interaction, 274–275, 275f
mechanical oscillators, 269–271, 270f
optical resonators, 272–274, 272f
Cavity protection, 154–155
Cavity quantum electrodynamics (QED), 441
with modified ONFs
Bragg resonators, 487–488
photonic crystal cavities, 487
Center-of-mass motion, BEC
decoupling of, 381
elimination of, 380–381
Chlorine (Cl\textsubscript{2}), dissociative electron attachment, 589–592, 590f
Circular rf polarization
in Ioffe–Pritchard trap, 214, 215f
in quadrupole trap, 219–221
Cisplatin, 554–555
Cladding-air guidance, 448–449
Clusters, dissociative electron attachment, 624–627
environmental influence on, 627–635, 632–634f
Coherent optical spectroscopy, 30
Cold target recoil-ion momentum spectroscopy, 509
Colinear geometry, 14
Collisions, electron, 547–548, 552–553, 555–556
Color-centers
in group-IV materials, 114–115
single photon emission with, 116–118, 117f
strong-coupling cavity QED with ensemble of, 157–161, 159–161f
COLTRIMS FDCS measurements, 537–538
Common optics approach, 18
Configuration-interaction resonance theory, 556–557
Constants of motion
of GP equation, 409–410
angular momentum of BEC, 414–415
energy of BEC, 412–413
generalized Ehrenfest theorem, 411–412
time-dependent TF approximation, 410–411
Hamiltonian formalism for matrix differential equation, 405–409
angular momentum conservation, 408–409
canonical form, 406
energy conservation, 408
equations of motion, 407–408
Continuous-wave (CW) excitation, 166
CO. See Carbon monoxide (CO)
Coupled quantum wells, 39–42
Coupled resonances, 6, 13–14
Coupling, 39
CP potentials. See Casimir–Polder (CP) potentials
Crossed-beam experiments, 566–570, 568f
Cross-section measurements, 566–570, 568f
CW excitation. See Continuous-wave (CW) excitation
D
Dark-count rate, 65–67
dCMP. See Deoxycytidine monophosphate (dCMP)
DCMs. See Dichroic mirrors (DCMs)
DD. See Dipolar dissociation (DD)
DDS. See Direct Digital Synthesis (DDS)
DEA. See Dissociative electron attachment (DEA)
de Broglie wavelength, 514, 537
Debye-Waller factor, 158
Decay of rf traps, estimates for, 244–249
Density matrix elements, 21
Deoxycytidine monophosphate (dCMP), 621–622, 623f
Detected-photon distribution, 70–72
Detuned photon blockade, in multiemitter cavity QED system, 168–169, 169f
Diagonal arrows, 21
Diamond
nitrogen-vacancy center in, 116–118
silicon vacancy in, 117–118, 117f
Dichroic mirrors (DCMs), 15
Diffractive optics, 18
Dipolar dissociation (DD), 580, 590–591, 638
Dipole–dipole interaction, 26
Direct-detection schemes, 65–67
Direct digital synthesis (DDS), 249, 251–252
Dissociative electron attachment (DEA), 548f
Arrhenius law, 575–578, 576f, 578f, 591–592
biologically relevant molecules, 609–623
extension to larger bio-/macromolecules, 621–623, 623f
formic acid, 610–614, 611f, 613f
Dissociative electron attachment (DEA)
(Continued)
formic acid dimer and larger clusters,
614–616, 615f
threshold cusps in electron attachment
to, 616–621, 617f, 619–620f
vibrational Feshbach resonance,
616–621, 617f, 619–620f
clusters, 624–635
environmental influence on
00115:s0200, 632–634f
van der Waals clusters, 624–627,
628–629f
to diatomics, 579–594
carbon monoxide, 582–584, 584f
halogen molecules, 589–594
hydrogen, 579–580, 579f
hydrogen halides, 585–589, 586f,
588f
oxxygen, 580–581, 582–583f
experimental methods, 565–575
crossed-beam experiments and
cross-section measurements,
566–570, 568f
velocity map imaging, 570–575, 571f,
573–574f
halogen-containing polyatomic
molecules, for plasma technologies,
594–600
interaction between resonances, 563–565,
565f
long-range effects, 557
low-energy behavior, 557–558
multidimensional vibrational dynamics in
local approximation, 601–609
acetylene, 607–609
hydrogen cyanide, 607–609
perfluoromethyl chloride, 602–605,
604f
water, 605–607
plasma technologies, 549–552, 552–553f
radiation chemistry, 552–555, 554–555f
threshold cusps in polyatomic molecules,
559–563, 560–562f
as universal phenomenon, 555–556
vibrational Feshbach resonance,
559–563
DNA, electron-induced structural damage
in, 552–553
Double-dressing, in multiple rf fields,
235–238
Double-quantum
cohereances, 24–26
wells, 39
Double-sided Feynman diagrams,
21–22, 21f
Dressed atom approach to magnetic
resonance with rf field, 200–203
dressed states in rotating wave
approximation, 203
rf coupling effects, 202–203
uncoupled states, 201–202
E
Effective Hamiltonian approach
to multimitter cavity QED, 161–165,
162f, 164–165f
to $n$-photon generation, 171–172, 171f
to photon blockade, 170, 170f
EIT. See Electromagnetically induced
transparency (EIT)
Electromagnetically induced transparency
(EIT), 443
Electromagnetic modes, in nanofiber
platform, 448–457
field equations, 449–450
higher-order modes, 453–456
light propagation, 453, 456–457
normalization, 451–453
propagation constant, 450–451
quasilinear polarization, 453–454
Electromagnetic radiation, 4
Electron collisions, 547–548, 552–553,
555–556
Electronic transitions, 2D coherent
spectroscopy of, 9–19
experimental implementation of,
14–19
multidimensional coherent spectroscopy
of atomic vapor
one-and zero-quantum 2D spectra,
19–22
three-dimensional coherent
spectroscopy, 26–29
two-quantum 2D spectra, 22–26
spectroscopy
linear spectroscopy, 4–6
nonlinear spectroscopy, 7–9
beyond transient four-wave mixing, 10–11
2D spectra, primer on, 12–14
Electron-induced chemistry, 548–549
Electron multiplying CCD (EMCCD) cameras, 65–67, 73–74, 95–96
Electron–phonon interaction, in solid-state cavity QED systems, 136–139, 137f
Electron–projectile velocity matching, 525–526
Electron scattering resonances, 547–548
Electron sponge, 597
Electrostatic parallel-plate analyzer, 511–512
EMCCD cameras. See Electron multiplying CCD (EMCCD) cameras
Emitter cavity detuning, in Jaynes–Cummings system, 131–135, 132f
Energy conservation, 408
Entanglement swapping, time-continuous, 314–317, 314f
conditional master equation, 315–316
feedback master equation, 316–317
optomechanical, 325–330, 328f
Equations of motion, Hamiltonian and, 407–408
Excitation beams, 14
Excitation pulse wavelength, 135–136, 136f
Excitons, 31
EXLPA method, 562–563, 568
Experimental two-quantum 2D spectra, 36–37

F
Fabrication and characterization, in nanofiber platform, 458–464
algorithm, 460
characterization and microscopy validation, 461
cleaning and alignment procedure, 460–461
fiber-pulling apparatus, 459–460
ONF radius extraction, 462–464
spectrogram analysis, 462
transmission, 461
Fabry–Pérot cavity, 270–272, 274–276, 487–488
FALP. See Flowing afterglow–Langmuir probe (FALP) method
Fano factor, 90–91, 100–101
Far-field phase shift, 5
FBP. See Few-body problem (FBP)
FDCS. See Fully differential cross sections (FDCS)
FEBIP. See Focused electron beam-induced processing (FEBIP)
Fedorov ratio, 63, 79–80
Feedback
Markovian homodyne, 354–356
master equation, 316–317
Fermi’s golden rule, 442–443
Few-body problem (FBP), 508
Fibercore SM1500 fiber, 456
Fiber–loop detector, 65–67
FIMMPROP, Maxwell’s equations solver, 461
Finite-difference time-domain (FTDT), 456
First Born approximation, 509–510, 532–533
Flowing afterglow–Langmuir probe (FALP) method, 591
Fluorine (F2) dissociative electron attachment, 589–592
Fluorocarbon plasmas, 550–551
Focused electron beam-induced processing (FEBIP), 638–639
Formic acid (HCOOH) dimer and larger clusters, 614–616, 615f
dissociative electron attachment, 610–614, 611f, 613f
Fourfold differential cross sections (4DCS), 535–536
Fourier transform, 2, 468–469, 530
Franck–Condon factor, 547–548, 580, 589, 591, 632–633
Franck–Hertz experiment, 546–547
Free expanding BECs, 415
long-time behavior of macroscopic wave function, 418–420
GP equation, 419–420
Schrödinger equation, 418–419
matrix differential equation, analytic solutions of, 415–418
Frequency-domain spectrum, 10–11
FTDT. See Finite-difference time-domain (FTDT)
Full microscopic theory, 34
Full width at half maximum (FWHM), 61, 442–443
Fully differential cross sections (FDCS), 509, 520–522
FWHM. See Full width at half maximum (FWHM)

G
GaAs
InAs quantum dots in, 113–114, 114f
quantum wells, 30
Gaussian profile, 12
Gaussian wave packet, 422, 528
Gauss’ law, 454
GEANT4-DNA, 554–555
Geiger ON/OFF mode, 90–91
Generalized Ehrenfest theorem, 411–412
Ghost imaging
bucket detector, 93
classical and quantum fields, 93
experimental setup, 94–95
high-order ghost imaging, 97–98
Global Warming Potentials (GWPs), 551, 600
Gross–Pitaevskii (GP) equation, 376–377, 419–420
affine transformation, BEC
affinely transformed wave function, 383–384
center-of-mass motion elimination, 380–381
classical trajectory interpretation, 385–386
linear transformation, scaling approach, 381–383
nonrotating trap, 384
settings of, 379–380
Group–IV materials, color–centers in, 114–115
GWPs. See Global Warming Potentials (GWPs)

H
Halogen–containing polyatomic molecules, for plasma technologies, 594–600
carbon tetrafluoride, 595–597, 595–596f
sulfur hexafluoride, 597–600, 598–599f
Halogen molecules, dissociative electron attachment in, 589–594
bromine (Br₂), 592
chlorine (Cl₂), 589–592, 590f
fluorine (F₂), 589–592
heteronuclear halogen molecules, 592–594, 593f
iodine (I₂), 592
Hamiltonian system, 29
beam-splitter, 277–278, 278f
cavity–optomechanical, 275–279, 278f
equations of motion, 407–408
formalism for matrix differential equation, 405–409, 421
angular momentum conservation, 408–409
canonical form of, 406
energy conservation, 408
equations of motion, 407–408
Harmonic trap, BEC in, 411–412
Hartree–Fock approximation, 34, 35f
HCN. See Hydrogen cyanide (HCN)
Heaviside step function, 10–11
Heisenberg–Langevin equation, 163–164, 342–343
Heisenberg uncertainty relation, 308–309
Helium coherence effects, 526–537
Helium nanodroplets, 631–632
Helmholtz coils, 513–514
Hemispherical electron monochromator (HEM), 566
HEM. See Hemispherical electron monochromator (HEM)
Hermite–Gauss (HG) mode, 455–456
Heteronuclear halogen molecules dissociative electron attachment, 592–594, 593f
Higher-order modes (HOMs), 454–456
Higher–order probe, in dressed atom trap, 240–244, 245f
High-resolution fiber profilometry, 454–455
Homogeneous two–level system, 10–11
Homogeneous width, 6
HOMs. See Higher–order modes (HOMs)
HPD. See Hybrid photodetector (HPD)
Hudson–Parthasarathy equation, 340–342
Hybrid photodetector (HPD), 65–67
Hydrodynamical approach, 420–421
Hydrogen
dissociative electron attachment, 579–580, 579f
Hydrogen cyanide (HCN)  
dissociative electron attachment, 607–609

Hydrogen halides  
dissociative electron attachment, 585–589, 586f, 588f

I  
ImagEM, Hamamatsu, Japan, 94–95
Imaging spectrometer (Lot Oriel), 68–69
InAs quantum dots, in GaAs, 113–114, 114f
InGaAs/GaAs quantum wells, 39–40
Inhomogeneous broadening, 6

Intense twin-beams (TWB)  
amplification distributions, 100–101
applications, 89–98
conditional states, 89–93
ghost imaging, 93–98
correlations, 56–58
detection distributions, 99–100
different regimes and detectors, introduction to, 65–67
laboratory of Quantum Optics, University of Insubria, 52–53
laser system, 67–68
multimode structure, 51
multimode twin beam
phase mismatch, effect of, 59–62
nonclassical correlations, measurements of, 85–89
experimental setup, 86
results, 87–89
parametric down-conversion (PDC), 50
photons per pulse, 52–53
quantum-mechanical formalism, 50
Schmidt modes, 62–65
single-photon regime, 50–51
spatio-spectral properties, 73–85
far-field coherence areas, 78–80
near-field to far-field coherence areas, 80–85
sub-Poissonian states, 52–53
twin-beam generation theory, 53–55
Intensity autocorrelation coefficient, 64–65
Intensity–intensity correlation measurements, 484

Interaction between resonances, 563–565, 565f

Intramolecular vibrational energy redistribution (IVR), 599–600, 614–616

Iodine (I₂)  
dissociative electron attachment, 592

Ioffe–Pritchard (IP) trap, 208–209, 209–210f, 212–218
circular rf polarization in, 214, 215f
isomagnetic surfaces, 213–214

Ionizing radiation, 552–555

Ion-pair formation (IPF), 638
IPF. See Ion–pair formation (IPF)
IP trap. See Ioffe–Pritchard (IP) trap

Isolated two-level resonances, 13–14

Isomagnetic surface
Ioffe–Pritchard trap, 213–214
role in rf-dressed atom trapping, 208–209, 209f

Isotropic case, 416–418

Isotropic trap, for 2D gas using quadrupole fields, 221–222

IVR. See Intramolecular vibrational energy redistribution (IVR)

J

Jahn–Teller distortion, 597

Jaynes–Cummings model/system
single-emitter cavity QED, 119–120, 127–154
dissipative structure, 128–131, 147f
electron–phonon interaction, 136–139, 137f
emitter cavity tuning, 131–135, 132f
excitation pulse wavelength, 135–136, 136f
key considerations, 127
quantum emitter, blinking of, 139–143, 141f, 143f

K

Kalman–Bucy filter, 356

Kalman filter(ing), 356–362
derivation of equations, 359–362
linear Langevin equations, derivation of, 358–359
Kalman filter(ing) (Continued)
for quantum control of optomechanical systems, 330–337
experimental setup, 332–334, 333f
genral considerations, 330–332, 331f
results, 334–337, 334–335f, 337f
KER. See Kinetic energy release (KER)
Kinematically complete experiments, 332–334, 333f
general considerations, 330–332, 331f
results, 334–337, 334–335f, 337f
L
Laboratory of Quantum Optics, University of Insubria, 52–53
Laguerre–Gauss (LG) mode, 455–456
Landau–Zener model/paradigm, 187–190, 188f, 563–564
Larmor radii, 567
Laser photoelectron attachment (LPA), 560–563, 568, 624
Laser system, 67–68
LEPTS code, 554–555
Levi-Civita symbol, 414
LG mode. See Laguerre-Gauss (LG) mode
Lindblad master equation, 281–283
Linear–quadratic–Gaussian (LQG) control scheme, 302–303, 362–364
Linear rf polarization
in Ioffe–Pritchard trap, 215–217, 216f
in quadrupole trap, 222–224
Linear spectroscopy, 4–7
Linear spectrum of inhomogeneous broadening, 6
LO. See Local oscillator (LO)
Loading from magnetic trap, 209–212, 211f
Local oscillator (LO), 65
Long-range electron–molecule interactions, 557
Long-time behavior of macroscopic wave function, 418–420
GP equation, 419–420
Schrödinger equation, 418–419
Long-time limit, 416
Low-energy behavior, 557–558
LPA. See Laser photoelectron attachment (LPA)
LQG control scheme. See Linear–quadratic–Gaussian (LQG) control scheme
M
Macroscopic wave function, long-time behavior of, 418–420
GP equation, 419–420
Schrödinger equation, 418–419
Magic wavelengths, 475–476
Magnetic field geometry, in quadrupole trap, 218–219
higher-order probe, in dressed atom trap, 240–244
Magnetic interaction, 194
Magnetic resonance theory to rf polarization, generalization of, 198–200
Magnetic trap, loading from, 209–212, 211f
Magneto-optical trap (MOT), 467
Magnification factor, 100–101
Markovian homodyne feedback, 354–356
Matrix differential equation
analytic solutions of, 415–418
Hamiltonian formalism for, 405–409
angular momentum conservation, 408–409
canonical form of, 406
energy conservation, 408
Hamiltonian and equations of motion, 407–408
Maxwell equations, 4, 449
MBE. See Molecular beam epitaxy (MBE)
Mean-field terms, 39
Mean photon-number value, 54
Mechanical oscillators, cavity–optomechanical systems, 269–271, 270f
MERT. See Modified effective range theory (MERT)
“Model” system, 3
Modified effective range theory (MERT), 557
Molecular beam epitaxy (MBE), 113–114
Mollow triplet spectrum, 468–469
Momentum distribution, 421–422
Monochromatic field modes, 53–54
Monte Carlo calculation, 525–526
MOT. See Magneto-optical trap (MOT)
Multidimensional coherent spectroscopy, 2–5
of atomic vapor
  one-and zero-quantum 2D spectra, 19–22
three-dimensional coherent
  spectroscopy, 26–29
two-quantum 2D spectra, 22–26
Multidimensional Fourier transform
  spectroscopy, 2
Multidimensional optical nonlinear
  spectrometer (MONSTR), 15–17
Multidimensional vibrational dynamics, in
  local approximation, 601–609
acetylene, 607–609
hydrogen cyanide, 607–609
perfluoromethyl chloride, 602–605, 604f
water, 605–607
Multimode cavity QED system, 154–165,
155f
  effective Hamiltonian approach to,
    161–165, 162f, 164–165f
  nonclassical light generation with, 166–172
detuned photon blockade, 168–169,
  169f
  effective Hamiltonian approach to
    n-photon generation, 171–172,
    171f
  effective Hamiltonian approach to
    photon blockade, 170, 170f
  resonant photon blockade, 166–167,
    167–168f
strong-coupling cavity QED with ensemble
  of color-centers, 157–161, 159–161f
Tavis–Cummings model, 155–157, 156f
Multimode twin beam
  phase-matched interaction, 59f
  phase mismatch, effect of, 59–62
  single-shot far-field pattern, 60–61
Multiple rf fields, 233–244
  double-dressing and rf evaporative
    cooling, 235–238, 238f
  higher-order probe, in dresses atom trap,
    240–244, 245f
  trap spectroscopy, 238–240, 238f, 241f
  well-separated rf frequencies, 233–235,
    234f
N
Nanofiber platform
  atoms and their interactions with ONFs,
    464–469
  atom–cloud characteristics, 467
  atomic dipole trap, 469–472
  atom–surface interactions, 465–467
dispersive measurements, 473–474
  ground state coherence, 476
  higher-order modes, trapping, 479
  linear and nonlinear spectroscopy,
    467–469
  possible heating mechanisms, 479–480
  reflectivity, 476–478
  state-sensitive and state-insensitive
    traps, 475–476
  transmission spectroscopy, 472–473
  trapping, 469–480
cavity QED, with modified ONFs
  Bragg resonators, 487–488
  photonic crystal cavities, 487
  chiral quantum optics, 445–446
  cooperativity in, 446–447
  electromagnetic modes, 448–457
  field equations, 449–450
  higher-order modes, 453–456
  light propagation, 453, 456–457
  normalization, 451–453
  propagation constant, 450–451
  quasi-linear polarization, 453–454
  emission enhancement parameter, 446
  fabrication and characterization, 458–464
  algorithm, 460
  characterization and microscopy
    validation, 461
cleaning and alignment procedure,
  460–461
  fiber-pulling apparatus, 459–460
  ONF radius extraction, 462–464
  spectrogram analysis, 462
  transmission, 461
  fiber optic technology, 445
heat-and-pull method, 445
nanophotonic systems, chirality in,
  489–492
in ONF systems, 491–492
  optical control of directionality in,
  490–491
Nanofiber platform (Continued)
optical nanofibers (ONFs), high cooperativity and optical depth
fiber optics, 447
optical dipole trapping of atoms, 447–448
science and engineering, use in, 447
Purcell effect, 446, 482
quantum optics and quantum information
antibunching, 484
atom light coupling, geometric dependence, 480–482
EIT and optical memories, 484–487
ONF-mediated coherent interactions, 483–484
Purcell effect, 482
waveguide coupling efficiency, 446
Nanophotonic systems, chirality in, 489–492
in ONF systems, 491–492
optical control of directionality in, 490–491
Neutral-density filters, 68
Newton sphere, 571, 571f
Nitrogen-vacancy (NV) center in diamond, 116–118
5-Nitroauracil, 554–555
NMR, 2
Noether’s theorem, 408–409
Noise reduction factor, 56
Nonadiabatic coupling, 186–187
Nonclassical light generation
basics of, 125–127, 126f
effects on quantum emitter blinking, 141–143, 143f
effects on self-homodyne interference, 148–153, 150f, 152f
with multiemitter cavity QED system, 166–172
detuned photon blockade, 168–169, 169f
effective Hamiltonian approach to n-photon generation, 171–172, 171f
effective Hamiltonian approach to photon blockade, 170, 170f
resonant photon blockade, 166–167, 167–168f
Nonlinear spectroscopy, 7–9
Nonunit detection efficiency, 56
Nonvacuum field states, 345–346
N-photon generation, effective Hamiltonian approach to, 171–172, 171f
Nuclear-excited Feshbach resonances, 559
O
OHT. See Optical homodyne tomography (OHT)
1D coherent optical spectroscopic techniques, 30
One-dimensional integrated density distribution, BEC, 391
One-quantum 2D spectra, 34
of excitons in GaAs quantum wells, 31–35
of quantum wells, 30
One-quantum spectrum, 17
Open system dynamics, of cavity-optomechanical systems, 279–286
master equation and stochastic master equation, 281–283
phase space description, 283–286, 285f
quantum Langevin equations, 280–281
Optical depth (OD) and cooperativity
decay rate, 441–442
electromagnetically induced transparency, 443
Fabry–Perot cavity, 442–443
Fermi’s golden rule, 442–443
full width at half maximum, 442–443
nanophotonic waveguide, 444
Rabi splitting, 441–442
strong coupling regime, 441–442
Optical excitations in semiconductors, 30–31
Optical homodyne tomography (OHT), 65
Optical nanofibers (ONFs), high cooperativity and optical depth
fiber optics, 447
optical dipole trapping of atoms, 447–448
science and engineering, use in, 447
Optical resonators, cavity-optomechanical systems, 272–274, 272f
Optical-spring effect, 282, 318
Optomechanical cooperativity, 279
Optomechanical feedback cooling, 301–306, 302f, 304–305f
Optomechanical systems, quantum control of, 301–306, 302f, 304–305f
cavity–optomechanical systems, 269–290 Hamiltonian, 275–279, 278f
open system dynamics, 279–286, 285f
optomechanical phase diagram, 286–290, 287f
physical system, 269–275, 270f, 272f, 275f
full dynamics, 297–301
perturbative dynamics, 298–299 results, 299–301, 300f
Kalman filter, implementation of,
330–337
experimental setup, 332–334, 333f
general considerations, 330–332, 331f
results, 334–337, 334–335f, 337f
optomechanical time-continuous teleportation, 317–325
pulsed entanglement, 290–301, 291f
creation of, 291–297
optomechanical teleportation protocol, 295–297, 296f
verification of, 294–295
time-continuous quantum control, 301–330
Bell measurements, 307–317
optomechanical feedback cooling, 301–306, 302f, 304–305f
optomechanical time-continuous entanglement swapping, 325–330, 328f
optomechanical time-continuous teleportation, 317–325, 320f
Optomechanical teleportation protocol, 295–297, 296f
Oxygen
dissociative electron attachment, 580–581, 582–583f
Oxyhydrogen flame, 459–460

P
Parametric approximation, 54
Parametric down-conversion (PDC)
in far-field coherence areas, 78–80
in ghost imaging, 93–98
nonlinear process, in uniaxial crystal, 59–60
pump field, evolution of, 51
theoretical investigation of, 50
in twin-beam generation, 53–55
PDC. See Parametric down-conversion (PDC)
Perfluoromethyl chloride
dissociative electron attachment, 602–605, 604f
Perturbative dynamics, of optomechanical systems, 298–299
Phase fluctuations, 18
Phase space description, 283–286, 285f
Photoexcitation, 30–31
Photoionization, 566, 568, 597–598
Photon echoes, 8–9
Photonic crystal cavities, 487
Photon-mediated interactions, 441
Photon-number statistics
experimental setup, 68–70
results, 70–73
Plasma(s)
fluorocarbon, 550–551
technologies, 549–552, 552–553f
halogen–containing polyatomic molecules for, 594–600
Poisson bracket, 407–408
Poissonian curves, 70–72
Polarization optics, 32–34
Polyatomic molecules, threshold cusps in,
559–563, 560–562f
Position-sensitive detector (PSD), 570–571
Potassium (K) atomic vapor, 19
POVM. See Positive–operator valued measure (POVM)
Power spectral density (PSD), 250–252
Poynting vector, 451–452
Projectile coherence
3-body distorted wave (3DW) approach, 531
cold target recoil-ion momentum spectroscopy, 509
COLTRIMS FDCS measurements, 537–538
de Broglie wavelength, 514, 537
electron–projectile velocity matching, 525–526
electrostatic parallel-plate analyzer, 511–512
Projectile coherence (Continued)
experimental methods, 511–514
few-body problem (FBP), 508
first Born approximation, 509–510, 532–533
fourfold differential cross sections (4DCS), 535–536
Fourier transform, 530
fully differential cross sections (FDCS), 509, 520–522
Gaussian wave packet, 528
H$_2$ coherence effects
Gaussian wave packet, 518–519
single and double differential studies, 515–520
Helmholtz coherence effects, 526–537
H$_2$ ionization, 510–511
ion impact ionization, 509
momentum vectors, 512
Monte Carlo calculation, 525–526
recoil-ion momentum, 513–514, 523, 525–526
Schrödinger equation, 508, 528
single-center interference, 522–524
target ionization, electron impact, 508–509
transfer ionization (TI), 526
wave packet scattering, 534
Projection–operator theory, 556–557
PSD. See Position–sensitive detector (PSD)
Pseudo–diatomic approach, 618
Pseudo–spectral (split-step) integration method, 75–76
ps–pulsed laser source, 52–53
Pulsed entanglement, 290–301, 291
creation of, 291–297
verification of, 294–295
Pulse–shaper approaches, 18
Purcell effect, 441, 446–447, 482

Q
QED. See Quantum electrodynamics (QED)
QLEs. See Quantum Langevin equations (QLEs)
QND interaction. See Quantum nondemolition (QND) interaction
QSDE. See Quantum stochastic differential equation (QSDE)
Quadratic phase matrix, 383
Quadrupole trap, rf-dressed, 218–224
circular polarization in, 219–221
isotropic trap for 2D gas using quadrupole fields, 221–222
linear polarization in, 222–224
magnetic field geometry in, 218–219
Quantum control, of optomechanical systems. See Optomechanical systems, quantum control of
Quantum control theory, 266–267
Quantum dots
strong coupling with, 118–119, 119
Quantum electrodynamics (QED), 441
multiemitter cavity (see Multiemitter cavity QED system)
single-emitter cavity (see Single-emitter cavity QED system)
Quantum emitter, blinking of, 139–143
nonclassical light generation effects, 141–143, 143
transmission spectra effects, 139–141, 141
Quantum estimation theory, 266–267
Quantum filtering, 266–267, 346–364
Belavkin equation, 351–354
in linear systems, 356–364
Kalman filtering, 356–362
LQG control, 362–364
Markovian homodyne feedback, 354–356
Quantum Langevin equations (QLEs), 280–281, 340
Quantum nondemolition (QND) interaction, 278–279
Quantum–optical model, 338–340
Quantum optics and quantum information
antibunching, 484
atom light coupling, geometric dependence, 480–482
EIT and optical memories, 484–487
ONF-mediated coherent interactions, 483–484
Purcell effect, 482
Quantum pressure, 389–390
Quantum state engineering, 98–100
Quantum state, stochastic evaluation of, 343–344
Quantum stochastic calculus, 338–346
Heisenberg–Langevin equation, 342–343
Hudson–Parthasarathy equation, 340–342
master equation, 344
nonvacuum field states, 345–346
quantum–optical model, 338–340
stochastic equations in Stratonovich form, 344–345
stochastic evaluation, of quantum state, 343–344
Quantum stochastic differential equation (QSDE), 338–340
quasi-Fock states, 89–90
generalized, 197–198, 204
Rabi splitting, 441–442
Radiation chemistry, 552–555, 554–555f
Radio frequency (rf)
adiabatic potentials for rf-dressed atoms, 203–212
Ioffe–Pritchard trap, 212–218, 216f
isomagnetic surface, role of, 208–209, 209–210f
loading from magnetic trap, 209–212, 211f
local coupling, 205–208
evaporative cooling, 235–238, 238f
magnetic resonance theory to, generalization of, 198–200
misalignment effects of, 254–256
multiple rf fields, 233–244
double-dressing and rf evaporative cooling, 235–238, 238f
higher-order probe, in dressed atom trap, 240–244, 245f
trap spectroscopy, 238–240, 240f, 241f
well-separated rf frequencies, 233–235, 234f
polarization in Ioffe–Pritchard trap circular, 214, 215f, 219–221 linear, 215–217, 216f
practical issues, 244–256
misalignment effects, of rf polarization, 254–256
rf stability and experimental conditions, 249–252, 250f
rf traps decay, estimates for, 244–249
rotating wave approximation, 253–254, 254f
quadrupole trap, 218–224
circular polarization in, 219–221
isotropic trap for 2D gas using quadrupole fields, 221–222
linear, 222–224
linear polarization, 222–224
magnetic field geometry in, 218–219
ring traps, 224–225, 225–226f
spin coupled to, 195–198
stability, 249–252, 250f
time-averaged adiabatic potentials, 226–233, 227f
well-separated, 233–235, 234f
Radiosensitizers, 554–555
Raman excitation scheme, 2
Ramsey fringes, 476
Rayleigh scattering, 453–454, 461, 463, 471, 486–487
Recoil-ion momentum, 513–514, 523, 525–526
Renner–Teller coupling, 601
Resonance surface, 205–206
Resonant frequencies, 6
Resonant photon blockade, in multiemitter cavity QED system, 166–167, 167–168f
Riccati equation, 284
Ring traps, 224–225, 225–226f
R-matrix method, 556–557, 581, 587, 589, 591, 603–605, 635
Root-mean-square (rms), 458–459
Rotating trap
affine approach application
2D model of BEC in, 396–397
free expansion after switching off, 399–403
time evolution within, 397–399
dressed states in, 203
practical issues in, 253–254, 254f
Rotation of spin operators, 192–193, 193f
Rotation operators, 191–192
Routh–Hurwitz criterion, 286
Rydberg atom, 441–443, 445

S
Scalar light shift, 475
Scaling approach, 376–377
linear transformation as natural generalization, 381–383
nonrotating trap, 384
Scaling parameters approach, 416
Scanning electron microscope (SEM), 461
Schmidt modes, 51, 62–65
stochastic, 308–310
Schwarz inequality, 57–58
Second-order intensity autocorrelation, 64–65
Self-homodyne interference (SHI)
in single-emitter cavity QED, 144–153, 147f
emission spectra effects, 146–148, 147f
nonclassical light generation effects, 148–153, 150f, 152f
Sellmeier dispersion, 75–76
SEM. See Scanning electron microscope (SEM)
Semiconductor double-quantum wells, 39
Semiconductor quantum wells, 2D
spectroscopy of excitons, 30
coupled quantum wells, 39–42
one-quantum 2D spectra of excitons in GaAs quantum wells, 31–35
optical excitations in semiconductors, 30–31
two-quantum 2D spectra of excitons in GaAs quantum wells, 35–39
Semiconductors, optical excitations in, 30–31
SF6
dissociative electron attachment, 551, 597–600, 598f
SHI. See Self-homodyne interference (SHI)
SiC. See Silicon carbide (SiC)
Signal-to-noise ratio (SNR), 96
Silicon carbide (SiC), 117–118
Silicon vacancy (SiV)
in diamond, 117–118, 117f
Single-emitter cavity QED system
Jaynes–Cummings model/system, 119–120, 127–154
dissipative structure, 128–131, 147f
electron–phonon interaction, 136–139, 137f
emitter cavity detuning, 131–135, 132f
excitation pulse wavelength, 135–136, 136f
key considerations, 127
quantum emitter, blinking of, 139–143, 141f, 143f
self-homodyne interference, 144–153, 144f, 147f, 150f, 152f
nonclassical light generation, basics of, 125–127, 126f
single photon emission with color-centers, 116–118, 117f
strong coupling, observing, 120–125, 121f, 124f
strong coupling with quantum dots, 118–119, 119f
Single-photon avalanche diodes (SPADs), 65–67
Single photon emission, with color-centers, 116–118, 117f
SiV. See Silicon vacancy (SiV)
SME. See Stochastic master equation (SME)
SNR. See Signal-to-noise ratio (SNR)
SPADs. See Single-photon avalanche diodes (SPADs)
Spatial density distribution, 421–422
Spatio-Spectral properties, in TWB, 73–85
Spectral hole burning, 7
Spectral interferometry, 17
Spectral pattern, 29
spectral resonances, 6
Spectroscopy
linear spectroscopy, 4–6
nonlinear spectroscopy, 7–9
Spin coupled to rf field, 195–198
Spin operators, 190–191
rotation of, 192–193, 193f
Spin states, 30
SSE. See Stochastic Schrödinger equation (SSE)
Stochastic master equation (SME), 281–283
Stochastic Schrödinger equation (SSE), 308–310
Stratonovich form, stochastic equations in, 344–345
Strong coupling
cavity QED with single quantum emitter, 120–125, 121f, 124f
with quantum dots, 118–119, 119f
Strong-coupling cavity QED with ensemble of color-centers, 157–161, 159–161f
Sub-Poissonian states, 52–53
Sulfur hexafluoride, 597–600, 598–599f

T
Target ionization, electron impact, 508–509
Tavis–Cummings model, 155–157, 156f, 172
Taylor’s series, 61–62
Teleportation
optomechanical protocol, 295–297, 296f
time-continuous, 307–314
conditional master equation, 308–310
feedback master equation, 311–314
optomechanical, 317–325, 320f
Temporary negative ions (TNIs), 547–548, 571–572
TEM. See Trochoidal electron monochromator (TEM)
TFWM. SeeTransient four-wave mixing (TFWM)
Thermionic emission, 566–568
Thomas–Fermi (TF) approximation, 376–377
3-body distorted wave (3DW) approach, 531
3DCS. See Three-dimensional coherent spectroscopy (3DCS)
Three-dimensional BEC
density distribution of, 390–391
Three-dimensional coherent spectroscopy (3DCS), 26–29
Three-dimensional density distribution,
BEC, 391
3D spectrum, 26–28
Three-pulse TFWM experiment, 10
Threshold cusps
in biological molecules, 616–621, 617f, 619–620f
in polyatomic molecules, 559–563, 560–562f
Time-averaged adiabatic potentials (TAAP), 226–233, 227f
Time-continuous quantum control, 301–330
Bell measurements
entanglement swapping, 314–317, 314f
teleportation, 307–314
optomechanical feedback cooling,
301–306, 302f, 304–305f
Time-dependent rotations, 193–194
Time-dependent TF approximation, 378–379, 409–411
BEC, macroscopic wave function in, 386–393
for dynamical evolution, 388–390
ellipsoidal density distribution boundaries of, 389f
for initial ground state, 386–387
integrated density distributions for
time-of-flight pictures, 390–391
rotating and vortex-free condensates, 392–393
Time-domain approaches of 2DCS, 14–15
Time-domain TFWM signal, 10–11
Time-integrated signal intensity, 9
Time-of-flight (TOF) mass spectrometer,
567, 568f, 570–571
Titromethane, 631–632
TNIs. See Temporary negative ions (TNIs)
TNT. See Trinitrotoluene (TNT)
TOF. See Time-of-flight (TOF) mass spectrometer
Traditional TFWM measurement, 9
Transfer ionization (TI), 526
Transient four-wave mixing (TFWM), 8–9
signals, 31
Transition–edge sensors (TES), 65–67
Transmission spectroscopy, 472–473
Trap spectroscopy, 238–240, 240f, 241f
Traveling-wave optical parametric amplifiers, 50
Trinitrotoluene (TNT), 631–632
Trochoidal electron monochromator (TEM), 567, 568f
2D coherent spectroscopy (2DCS) of electronic transitions, 9–19
beyond transient four-wave mixing, 10–11
2D spectra, primer on, 12–14
experimental implementation of, 14–19
multidimensional coherent spectroscopy of atomic vapor
one- and zero-quantum 2D spectra, 19–22
three-dimensional coherent spectroscopy, 26–29
two-quantum 2D spectra, 22–26
spectroscopy
linear spectroscopy, 4–6
nonlinear spectroscopy, 7–9
Two-dimensional integrated density distributions, BEC, 390–391
2D model of BEC, in rotating trap, 396–397
2D spectra, 26–28
primer on, 12–14
2D spectroscopy of excitons, semiconductor quantum wells, 30
one-quantum 2D spectra of excitons in GaAs quantum wells, 31–35
optical excitations in semiconductors, 30–31
Two-exciton states, 35–36
Two-mode squeezing (TMS) interaction, 277–278, 278f
Two-quantum coherence, 22–24
Two-quantum 2D spectra, 14, 19, 22–26
of excitons in GaAs quantum wells, 35–39
Two-quantum 2D spectroscopy, 26
Two-quantum signal, 26

V
VAEs. See Vertical attachment energies (VAEs)
van der Waals (vdW)
dissociative electron attachment clusters, 624–627, 628–629f, 636
potentials, 465
Velocity map imaging (VMI), 570–575, 571f, 573–574f, 581, 583–584, 592, 594–595, 599–600
Velocity slice imaging (VSI), 571–572, 574–575, 580, 585, 590–592, 594–595, 599–600
Vertical attachment energies (VAEs), 564–565
VFR. See Vibrational Feshbach resonance (VFR)
Vibrational excitation cross sections, threshold peaks in, 585
Vibrational Feshbach resonance (VFR), 557, 559–563, 585, 605–606, 611
in biological molecules, 616–621, 617f, 619–620f
Virtual factories, 550–551
Visibility (VIS), 96
Visible light photon counter (VLPC), 65–67
VLPC. See Visible light photon counter (VLPC)
VMI. See Velocity map imaging (VMI)
von Neumann entropy, 91–92
VSI. See Velocity slice imaging (VSI)

W
Water
dissociative electron attachment, 605–607
Waveguide coupling efficiency, 446
Wave packet scattering, 534
WGM. See Whispering-gallery-mode microresonator (WGM)
Whispering-gallery-mode microresonator (WGM), 490–491
Wigner cusps, 611
Wigner representation, 75–76
Wigner threshold law, 558, 589

Z
Zakai equation, 308–309
Zero-phonon line (ZPL), 116–118
extraction ratio, 158
Zero-quantum 2D spectra, 14, 19–22, 28, 40
Zero-quantum spectrum, 17
ZPL. See Zero-phonon line (ZPL)