Hybrid Homodyne-like Detection Scheme with Photon-Number-Resolving Detectors

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Abstract— The measurement of quantum states of light is fundamental for the investigations of Quantum Optics and for the applications of Quantum Information. Here we present the implementation of a novel detection apparatus, that is a homodyne-like detection scheme based on photon-number-resolving detectors. Thanks to its hybrid nature, the apparatus is useful to investigate both the particle-like and wave-like properties of light. We tested the scheme in the measurement of the quadratures of different optical states, namely coherent states and mixtures of coherent states. In particular, we investigate the performance of our scheme compared to that of a standard homodyne detector.

1. INTRODUCTION

The measurement of quantum states of light is of fundamental relevance not only for their characterization, but also for their exploitation in applications to Quantum Optics and Quantum Information. Essentially, there are two kinds of detection strategies, which give access to the complementary aspects of light: optical homodyne tomography (OHT) and direct detection. OHT investigates the wave-like properties of the states, while direct detection, which involves photon-number resolving (PNR) detectors, is used to explore the particle-like features of light.

From the technical point of view, OHT is based on an interferometric scheme in which the signal is mixed with a high-intensity local oscillator (LO), that is a coherent state with variable phase. The two outputs of the interferometer are detected by two pin photodiodes, whose difference photocurrent is suitably amplified and recorded as a function of the LO phase. By properly processing the data and applying a complex reconstruction algorithm, it is possible to achieve the complete knowledge of the signal.

As to direct detection, it is worth noting that many solutions exist to realize PNR detectors, even if the perfect detector is still missing. Among PNR detectors we mention: fiber-loop detectors, visible-light photon counters, transition-edge sensors and superconductive nanowires, hybrid photodetectors (HPDs) and Si-photomultipliers (SiPM). The latter two have the advantage of a quite easy operation at room temperature and, although being affected by imperfections, such as a limited dynamic ranges, rather low repetition rates (HPDs), dark count and cross talk effects (SiPM) and limited quantum efficiencies (about 50%), they have been already used to measure the quantum features of light [1, 2]. Recently, new hybrid schemes addressing both wave-like and particle-like aspects of light and operating in the low intensity regime have been proposed combining homodyne detection and direct detection [3–6]. Here we present the implementation of a homodyne-like detection scheme, in which a low-energy local oscillator (LO) and two hybrid photodetectors are used instead of the traditionally-employed high-energy LO and pin photodiodes. At variance with other existing homodyne-like detection schemes [7–9], the employment of HPDs as the PNR detectors allows us to explore a wide photon-number dynamic range (up to 30 photons). For this reason, the detection apparatus, though having an upper limit to the measurable light intensity, can be also useful to investigate different regimes of local oscillator (LO) intensity. We tested the scheme in the measurement of the quadratures of a number of different optical states, such as coherent states and mixtures of coherent states.

2. THEORY

In a standard homodyne scheme, the signal mode $\hat{a}$ interferes at a balanced BS with the LO mode $\hat{b}$ excited in a coherent state $|z\rangle$ with amplitude β ($\beta \in \Re$). The interference field is measured at the two BS outputs by two identical photodetectors, whose photocurrents are electronically processed to get the difference photocurrent

$$\Delta\hat{I} = \frac{\hat{I}_c - \hat{I}_d}{\sqrt{2}\beta} = \frac{\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger}{\sqrt{2}\beta}, \tag{1}$$

in which c and d identify the two BS outputs. If we assumethat the LO is excited in a strong semi-classical state, it is possible to neglect the LO fluctuations and operate the following substitutions: $\hat{b} \to \beta e^{i\phi}$ and $\hat{b}^\dagger \to \beta e^{-i\phi}$. Equation (1) then reduces to

$$\Delta\hat{I} = \frac{\hat{a}^\dagger \beta e^{i\phi} + \hat{a}\beta e^{-i\phi}}{\sqrt{2}\beta} = \frac{\hat{a}^\dagger e^{i\phi} + \hat{a} e^{-i\phi}}{\sqrt{2}} \equiv \hat{x}_\phi, \tag{2}$$

in which $\hat{x}_\phi$ represents the quadrature operator associated to the signal mode. In this limit, the probability distribution of the difference photocurrent approaches the probability distribution $p_\phi(x;\alpha) = {}_\phi\langle x|\rho|x\rangle_\phi$ of the quadrature $\hat{x}_\phi$. For instance, in the case of a signal mode $\hat{a}$ excited in the coherent state $|\alpha\rangle$, the homodyne distribution

$$p_{\mathrm{HD}}(x;\alpha) = p_\phi(x;\alpha) = \frac{1}{\sqrt{\pi}} \exp\left[-(x - \sqrt{2}\alpha\cos\phi)^2\right] \tag{3}$$

is a normal distribution with mean $\langle x\rangle = \sqrt{2}\alpha\cos\phi$ and variance $\sigma_x^2 = 1/2$.

At variance with the standard homodyne scheme, the homodyne-like detection scheme based on PNR detectors offers the possibility to have direct access to each BS output. For the case described above, in which the signal mode is excited in the coherent state $|\alpha\rangle$, it can be demonstrated that each output is described by a Poisson distribution of the number of photons, whose mean value is either $\mu_1 = (\alpha^2 + \beta^2)/(2) + \alpha\beta\cos\phi$ or $\mu_2 = (\alpha^2 + \beta^2)/(2) - \alpha\beta\cos\phi$ depending on which output is considered.

The distribution of the difference d of photon numbers can be evaluated as the convolution of the two Poisson distributions and is described by the Skellam distribution [10]

$$p_{\mathrm{DD}}(d;\alpha) = S(d;\mu_1,\mu_2) = e^{-\mu_c-\mu_d}\left(\frac{\mu_1}{\mu_2}\right)^{d/2} I_d(2\sqrt{\mu_1\mu_2}), \tag{4}$$

in which $I_d(x)$ is the modified Bessel function of the first kind. In the highly excited LO regime, in which $\beta >> |\alpha|$, the Skellam distribution describing the distribution of the difference of photon numbers well approximates the homodyne distribution. In particular, it can be demonstrated that, by rescaling the variable d as $d = x\sqrt{2}\beta$, $p_{\mathrm{DD}}(d = x\sqrt{2}\beta;\alpha) \to p_{\mathrm{HD}}(x;\alpha)$ for $\beta \to \infty$. The results obtained so far for a coherent state as the signal can be extended to the case of mixtures of coherent states, such as the so-called bracket states, already described in Ref. [11]. In particular, in the case of a bracket state described by the noise parametr γ, the distribution of the difference of photon numbers reads as

$$S(d;\mu_1,\mu_2,\gamma) = \int_{-\gamma/2}^{\gamma/2} \frac{d\phi}{\gamma} S(d;\mu_1,\mu_2). \tag{5}$$

3. EXPERIMENTAL SETUP

The homodyne-like detection scheme we realized is shown in Fig. 1. To test its performance, we exploited the scheme in the reconstruction of the distribution of the difference of photon numbers of coherent states and mixtures of them. As shown in Fig. 1, the second-harmonic pulses (5-ps-pulse duration) emitted at 523 nm by a mode-locked Nd:YLF laser regeneratively amplified at 500 Hz were sent to a Mach-Zehnder interferometer to get the signal and the LO. Two variable neutral density filters inserted in the two arms of the interferometer were used to change the balancing between the two fields. The spatial and temporal superpositions of signal and LO were optimized in order to get the best possible overlap admitted by the choice of the amplitudes and of the balancing.

The length of one of the two arms of the interferometer was changed in steps by means of a piezoelectric movement in order to modify the LO phase in the whole 2π-range. We set 60 different piezo positions and for each one we recorded 50000 laser shots.

The light at the two outputs of the second BS was collected by two multi-mode fibers (600-µm-core diameter) and sent to two HPDs (mod. R10467U-40, Hamamatsu). The output of each detector was amplified (preamplifier A250 plus amplifier A275, Amptek), synchronously integrated over a 500-ns window (SGI, SR250, Stanford) and digitized (AT-MIO-16E-1, National Instruments). Two typical pulse-height spectra of the HPDs are shown in the left panel of Fig. 2, in which the partial photon-number resolving capability of the detector is clear. Its response can be directly compared to those of SiPM detectors, which are shown in the right panel of the same figure. By

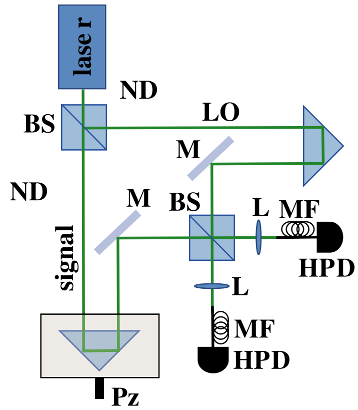

Figure 1: Sketch of the experimental setup. BS: beam splitter; ND: variable neutral density filters; Pz: piezoelectric movement; MF: multi-mode fibers; HPD: hybrid photodetectors; L: converging lenses.

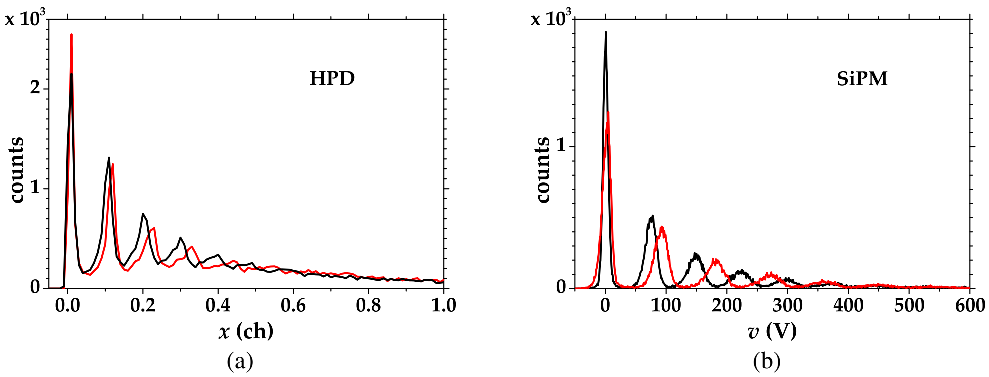

Figure 2: (a) Typical pulse-height spectra of two different HPDs. (b) Typical pulse-height spectrum of two SiPM.

applying the self-consistent model presented in Refs. [12, 13], we processed each BS output in order to reconstruct the distribution of detected photons.

Typical reconstructions of the photon-number statistics registered by the detectors are shown in Fig. 3, from which it is possible to appreciate the wide dynamic range of HPDs. We notice that the theoretical Poisson distributions are well superimposed to the experimental data with a very high fidelity (see values in the figure). This confirms the correctness of the model described in Section 2. As already explained in Refs. [11, 14], the linearity of HPDs allows the extraction of information about the phase. In fact, the mean number of photons detected at each output of the interferometer followes the interference pattern as a function of the piezo position. Thus, the relative phase between the two arms of the interferometer can be retrieved by normalizing the mean values between -1 and 1, and applying the arcos function. In Fig. 4 a typical result of a measurement is displayed. The monitoring of the mean values also allowed us to check the stability

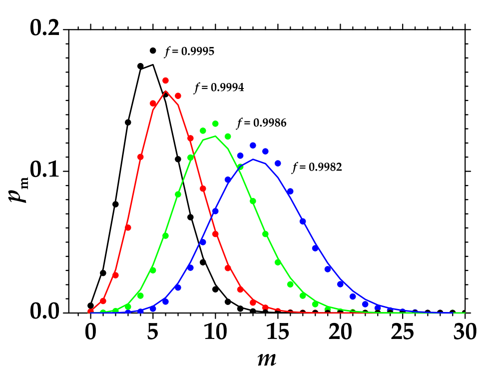

Figure 3: Reconstructed statistics at the output of one HPD for measurement on a Poissonian state.

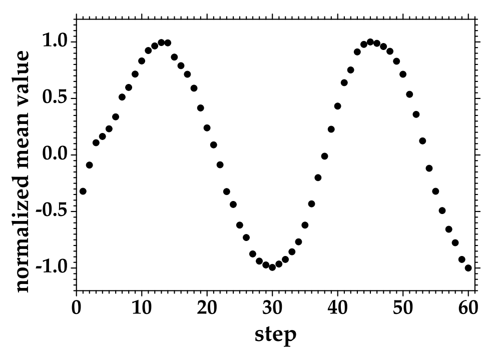

Figure 4: Normalized mean value of detected photons at the output of one of the detectors.

of the detection apparatus during the long measurement sessions.

Once assigned a phase $\phi \in (0, 2\pi]$ to each piezo position, by properly combining the data corresponding to the different phase values, it is also possible to build the so-called bracket states, which are mixtures of coherent states [15]. Such states are particularly useful to simulate the effect of uniform phase noise in communication channels with coherent states.

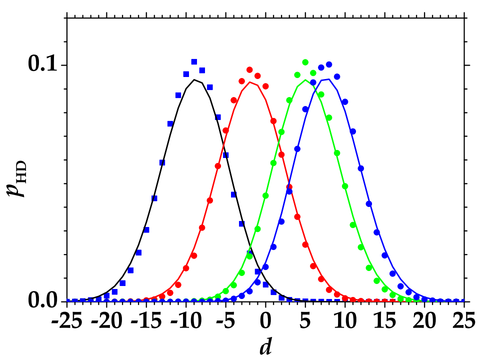

Figure 5: Distribution of the difference of photon numbers obtained from the experimental data in the case of a coherent state as the signal for different choices of the relative phase, ϕ.

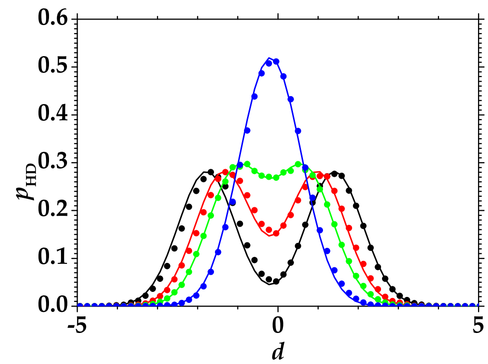

Figure 6: Distribution of the difference of photon numbers obtained from the experimental data in the case of a mixture of coherent states with opposite phases for different values of ϕ.

As anticipated in Section 2, by exploiting the homodyne-like detection scheme it is possible to have access to the difference of photon numbers, whose distribution is connected to the signal quadrature. In Fig. 5 we show the distribution of the difference of photon numbers obtained from the experimental data in the case of a coherent state as the signal for different choices of the relative phase, ϕ. Superimposed to the data, are the theoretical expectation given by Eq. (4). We note the superposition is quite good, even if with a modest unbalancing value (1:9) between signal and LO. Similar results can be achieved by considering as the input signal the sum of two coherent states with opposite phases (see Fig. 6).

4. CONCLUSION

In conclusion, we presented the implementation of a homodyne-like detection scheme employing PNR detectors instead of pin photodiodes and a low-intensity LO instead of a macroscopic one. We demonstrated that such a scheme is useful to have access both to the number of photons and to the wave-like properties of the states under investigation. In particular, we showed that by exploiting the linearity of the detection chain is possible to get information about the relative phase between signal and LO. Moreover, we calculated the difference of the photon numbers detected at the output of the beam splitter, which is related to the quadratures of the states. The good quality of the experimental results compared to the theoretical expectations suggests the possible exploitation of the detection scheme for the development of protocols based on coherent states.

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