

Effects of pump depletion on spatial and spectral properties of parametric down-conversion

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ABSTRACT

It is well known that optical twin-beam states (TWB) generated by spontaneous parametric down-conversion (PDC) exhibit spatial and spectral correlations, which can appear in single-shot images obtained by using an imaging spectrometer to resolve emission angles and wavelengths simultaneously. By analyzing series of single-shot images recorded by an EMCCD camera at different powers of the pump beam, we studied the evolution of several quantities characterizing the generated TWB. In particular, we demonstrated that correlation widths in spectrum and space increase monotonically at low pump powers and then start decreasing at higher powers due to the onset of pump depletion. In a complementary way, the Fedorov ratio decreases and then increases again. At the same time, the number of modes evaluated from photon statistics follows a complementary behavior to correlation widths that can be interpreted in terms of the evolution of the number of Schmidt modes in the field.

Keywords: Parametric down conversion, coherence, Fedorov ratio, Schmidt decomposition

1. INTRODUCTION

It is well known that optical twin-beam states (TWB) generated by spontaneous parametric down-conversion (PDC) exhibit spatial and spectral correlations. The first experimental measurements of spatial correlations in TWB, aimed at determining the size of the coherence areas in a transverse plane, were performed in the single-photon regime on twin-beam states containing one photon at most by using scanned single-photon detectors.¹⁻⁷ The experimental results can be compared to a well-established theory properly working in this regime, in which, for instance, the pump beam can be considered as non-evolving during the interaction (undepleted-pump approximation). On the other hand, TWB can also be generated in a much higher intensity regime,⁸⁻¹¹ in which coherence areas become visible in single-shot images at the output of the nonlinear crystal.

We used an EMCCD camera, combined to an imaging spectrometer that resolves emission angles and wavelengths simultaneously,¹² to record far-field single-shot images of the PDC emission. The system has already been exploited to characterize several properties of PDC, such as their dependence on pump power and tuning angle.^{12,13} We demonstrated that correlation widths in spectrum and space increase monotonically at low pump powers and then start decreasing at high pump powers due to the onset of pump depletion. At the same time, we demonstrated that the number of modes evaluated from photon statistics follows a complementary behavior

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with respect to correlation widths. This effect can be interpreted in terms of the evolution of the number of Schmidt modes in the radiation field. Note that in the pump-depletion regime the evolution of the pump field must be taken into account together with the evolution of the PDC.

In this paper we investigate the characterization of TWB properties in terms of their mode structure, by measuring the Fedorov ratio and the number of modes from photon statistics.

2. FEDOROV RATIO AND SCHMIDT NUMBER

The degree of entanglement of the TWB states generated at the single-photon level, and described by the so-called biphoton function, can be quantified by the parameter F defined as the ratio of widths of single-particle ($\Delta\mathbf{p}$) and coincidence wave packets ($\delta\mathbf{p}$)¹⁴

$$F = \frac{\Delta\mathbf{p}}{\delta\mathbf{p}} = \frac{\Delta\mathbf{q}}{\delta\mathbf{q}}, \quad (1)$$

being \mathbf{p} and \mathbf{q} the transverse wavevectors of signal and idler, respectively. It can be demonstrated¹⁶ that for double-Gaussian bipartite states the parameter F coincides with the Schmidt number K , defined as the inverse of the purity of the state of each separate subsystem

$$K = \frac{1}{Tr[\rho_1^2]}. \quad (2)$$

Note that K is another entanglement quantifier.¹⁵ It can be demonstrated that also in more realistic cases in which the biphoton function is not double-Gaussian, the parameters F and K are very close to each other.¹⁴

The value of the Fedorov ratio can be rather easily evaluated from coincidence measurements, while the Schmidt number can be obtained from the theory of the mode decomposition, in which the biphoton function is expressed as a sum of factorized terms^{15,17,18}

$$|\Psi_{12}\rangle = \sum_k \lambda_k |u_k\rangle |v_k\rangle. \quad (3)$$

Here $|u_k\rangle$ and $|v_k\rangle$ represent the eigenvectors of the orthonormal dual basis of the Schmidt modes. Eigenvalues λ_k of the decomposition give the probabilities p_k of detecting a photon in k -th mode, $p_k = \lambda_k^2$. The Schmidt number is defined as

$$K = \frac{1}{\sum_k \lambda_k^4}. \quad (4)$$

It can be demonstrated that the Schmidt number is connected to the second-order autocorrelation function, $g^{(2)}$, by

$$g^{(2)} = 1 + \frac{1}{K}, \quad (5)$$

which can be used for the determination of number K of modes.

In the case of more intense TWB, a quantity analogous to the Schmidt number can be evaluated in the same way by calculating the second-order intensity autocorrelation.^{12,19-21} Similarly, we can define a quantity F_θ for the macroscopic TWB in analogy to Eq. (1) by taking the ratio of the angular size of the downconverted light to the angular size of the cross-correlation area:

$$F_\theta = \frac{\Delta\theta}{\delta\theta}. \quad (6)$$

We could also define a similar quantity for spectral size of PDC, $F_\lambda = \Delta\lambda/\delta\lambda$, where $\Delta\lambda$ is the bandwidth of the generated PDC and $\delta\lambda$ is the spectral width of the cross-correlation area. However, we will not discuss this quantity here, because our experimental apparatus is able to register only a wavelength interval and not the entire PDC spectrum.

In the following we show the experimental results for the two quantities F_θ and K in the macroscopic regime up to pump depletion with the aim of investigating if they behave similarly. Note that the values of these quantities cannot be taken as a measure of entanglement as in the case of the single-photon regime.

3. EXPERIMENTAL SETUP

The experimental setup we used is shown in Fig. 1. The PDC process was generated by pumping a type-I 8-mm-long β -Barium-Borate (BBO) crystal (cut angle = 37 deg) with the third-harmonic pulses (349 nm, 4.5-ps pulse duration) of a mode-locked Nd:YLF laser (High-Q-Laser), regeneratively amplified at 500 Hz.

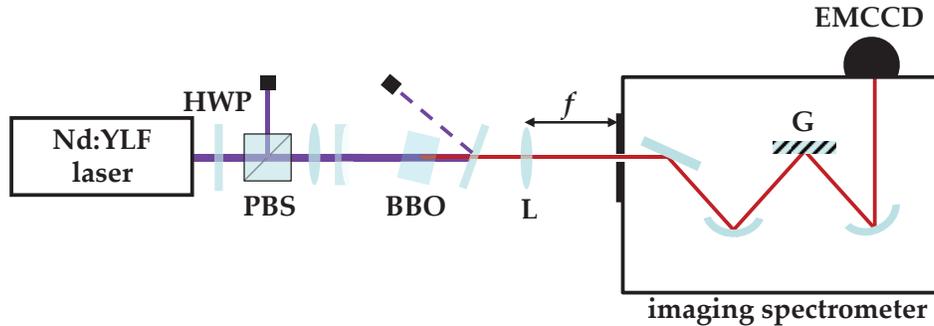


Figure 1. Sketch of the experimental setup used for the spatio-spectral measurements of TWB. HWP: half-wave plate; PBS: polarizing cube beam splitter; BBO: nonlinear crystal; L: lens, with 60-mm focal length; G: grating; EMCCD: electron-multiplying CCD camera.

The pump beam was collimated by means of a telescope in front of the BBO to a full width at half maximum (FWHM) of $\sim 380\mu\text{m}$ at the lowest pump power. The pump mean power was changed during the experiment by a half-wave plate followed by a polarizing cube beam splitter. The tuning angle was chosen to have phase-matching at frequency degeneracy in quasi-collinear configuration. The PDC light was collected by a 60-mm focal length lens and focused on the plane of the vertical slit of an imaging spectrometer (Lot Oriel) having a 600 lines/mm grating. The entrance slit of the spectrometer was thus located in the far-field plane of the PDC

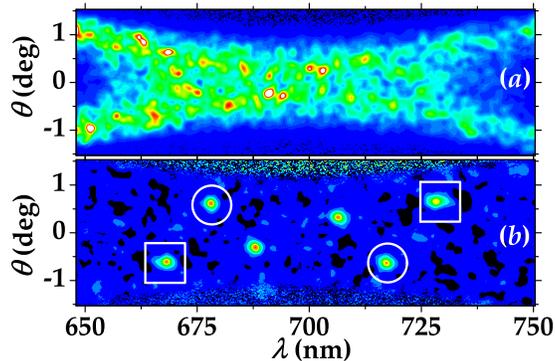


Figure 2. (a) Typical far-field spectrum registered at the output of the imaging spectrometer; (b) Typical example of matrix of intensity correlation coefficient $\Gamma_{k,l}^{(i,j)}$, in which the intensity auto- and cross-correlation areas are clearly evident.

field and the imaging spectrometer produced a frequency-dispersed image of the far field at the exit plane. The image was then recorded in single shot by a synchronized EMCCD camera (iXon Ultra 897, Andor), operated at full frame resolution (512x512 pixels, 16- μm pixel size). The resolution of the system composed of the imaging spectrometer and the EMCCD camera was 0.2 nm in spectrum and 0.015 deg in angle. We note that this resolution is much higher than that achievable by spectral filtering with interference filters and spatial selection with pin-holes.

In Fig. 2(a) we show a typical speckle-like pattern recorded by the EMCCD camera. Note that even in single-shot images we have evidence of coherence (in both spectrum and space) and of the presence of intensity correlations between the signal and idler portions of the twin beam. The exact shape of the far field pattern, and hence the characteristics of the PDC light, depend on pump spatial and spectral widths,¹³ crystal length,²² distance from BBO²³ and pump power.¹² Here we will focus our attention on Fedorov ratio and Schmidt number as a function of pump power.

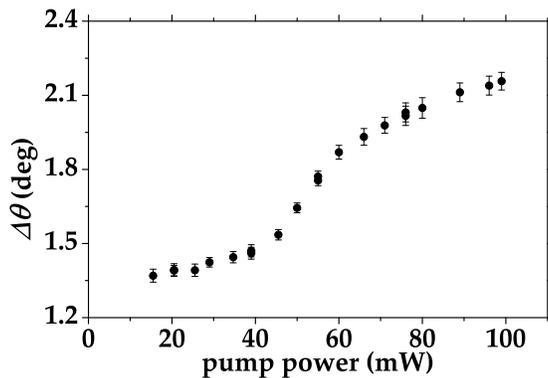


Figure 3. Spatial size of the spectrum (FWHM) taken around frequency degeneracy (698 nm) in quasi-collinear configuration as a function of pump mean power.

In order to calculate the values of F_θ for our system, first of all we experimentally determined the behavior of the spatial bandwidth of the PDC as a function of pump power (see Fig. 3). The bandwidth was calculated as the FWHM of the vertical section of the average far-field spectrum in correspondence to a number of different pixels close to frequency degeneracy in quasi-collinear direction. In this picture and in the following, error bars are determined from the variance of the values obtained for the different pixels. We see that the data have a nontrivial trend and definitely change behavior in correspondence of a value of the pump at which pump depletion becomes relevant (see below).

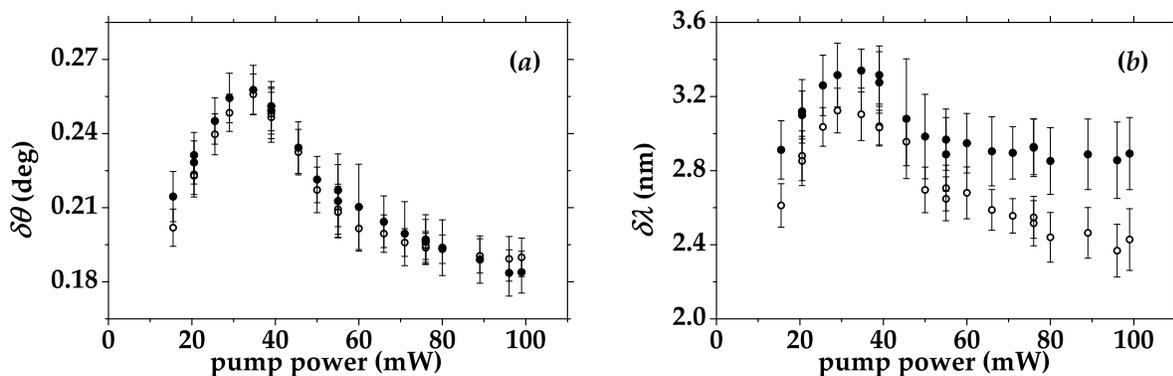


Figure 4. Evolutions of the spatial (a) and spectral (b) FWHM size of the intensity autocorrelation (empty circles) and cross-correlation (full circles) areas measured from far-field spectra of the TWB as functions of the pump mean power.

The second quantity required to evaluate F_θ is the size of the cross-correlation area. The areas were obtained by calculating the intensity correlation coefficient between a single pixel at coordinates (i, j) and all the pixels (k, l) contained in a single image, subtracted of the mean value of the noise measured with the camera in perfect

dark:

$$\Gamma_{k,l}^{(i,j)} = \frac{\langle I_{i,j} I_{k,l} \rangle}{\langle I_{i,j} \rangle \langle I_{k,l} \rangle}. \quad (7)$$

Here I is the intensity value of each pixel expressed in digital numbers and $\langle \dots \rangle$ indicates the average over 1000 subsequent images. Note that $\Gamma_{k,l}^{(i,j)}$ is a matrix having the same size as the original images that contains both the intensity auto- and the cross-correlation areas.

The choice of calculating correlations by starting from the intensity value in a single pixel, instead of that in a larger area, is motivated by the search for the closest analogy to single-photon measurements, in which correlations are calculated by counting coincidences between the outputs of two single-photon detectors, one of which is kept fixed in a certain position and the other is moved.

A typical matrix of intensity correlation coefficients is shown in Fig. 2(b), where the autocorrelation areas are on the left, and the corresponding cross-correlation areas are on the right (in the figure, three different choices of the starting pixel are displayed simultaneously). The horizontal section of these areas is related to spectrum and the vertical section to the angular dispersion. In Fig. 4 we show the FWHM widths in spectrum (a) and space (angle) (b) of the intensity autocorrelation (empty circles) and cross-correlation areas (full circles), as functions of the input pump mean power. In both panels, we can observe an initial growth that reaches the maximum at a pump power of about 30 mW and then decreases. As we have already demonstrated elsewhere,^{12,20} the behavior of correlation areas can be explained by taking into account that at high pump power values the strong nonlinear process leads to pump depletion, so that also the pump beam evolves in the nonlinear interaction. This makes the dynamics of the system more complex and affects the number of effectively populated signal and idler radiation modes. In fact, as the pump power increases, the PDC gain profile becomes narrower and narrower, and thus signal and idler fields are dominantly emitted into a smaller and smaller number of modes that gain energy and grow in size to the detriment of the others.^{19,24} For sufficiently high values of the pump power, the process of mode selection reverts as the pump profile undergoes depletion, producing a narrowing of the intensity autocorrelation and cross-correlation areas.

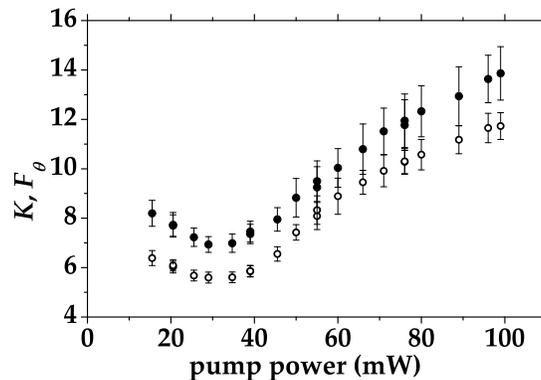


Figure 5. Fedorov ratio, F_θ , (full circles) and Schmidt number, K , (empty circles) as a function of the pump mean power.

In Fig. 5 (full circles) we show the value of the Fedorov ratio, F_θ , obtained by evaluating the ratio between the data for the spatial bandwidth (see Fig. 3) and those for the spatial cross-correlation areas (see full circles in Fig. 4(a)), as a function of the pump mean power. The behavior of the experimental data shows a trend with a minimum corresponding to the onset of pump depletion.

To compare the behavior at high gain of the Fedorov ratio F_θ to that of the Schmidt number K obtained from Eq. (5), we calculated the number of modes in the statistics of signal from the values of the maximum of intensity autocorrelation coefficient, that is by taking $g^{(2)} = \Gamma_{i,j}^{(i,j)}$. We remind that, in order to exploit the optimal resolution of our detection system, the number of modes K was obtained by calculating the autocorrelation

coefficient between a single pixel and all the pixels contained in the entire image for all the values of the pump mean power. In Fig. 5 we plot the resulting values for K (empty circles) together with those of F_θ .

The two quantities are not perfectly equal as expected from theory in the single-photon regime and for double-gaussian biphoton function. Nevertheless, they display the same trend and a nearly proportional behavior.

4. CONCLUSIONS

We have demonstrated that by extending the concepts of Fedorov ratio and Schmidt number from the microscopic to the macroscopic regime of TWB generation we obtained a characterization of the PDC emission that is consistent with a description of the evolution of the mode structure as a function of pump power. The extension seems reasonable in view of the experimental results, even if more theoretical work is still needed to fully explain them.

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