Real-time phase-reference monitoring in a quasi-optimal coherent-state receiver

Alessia Allevi[a], Matteo Bina[b], Maria Bondani[c], and Stefano Olivares[b]

[a]Department of Science and High Technology, University of Insubria and CNISM UdR Como, Via Valleggio 11, I-22100 Como, Italy;

[b]Department of Physics, Milano University and CNISM UdR Milano Statale, Via Celoria 16, 20133 Milano, Italy;

[c]Institute for Photonics and Nanotechnologies, CNR and CNISM UdR Como, Via Valleggio 11, I-22100 Como, Italy

ABSTRACT

The Kennedy-like receiver is a quasi-optimal discrimination scheme employed in binary phase-shift-keyed communication schemes with coherent states. In its standard configuration, it is based on the interference of the two signals encoding the message with a reference local oscillator and on the measurement by means of ON/OFF detectors. Here we demonstrate that, without interrupting the communication, it is possible to monitor the relative phase between the signals and the local oscillator by applying a Bayesian processing to the very data sample used to discriminate the signals at any shot. We show, both numerically and experimentally, that the minimum uncertainty in phase estimation can be achieved both with ON/OFF and photon-number resolving detectors. The performances of our phase-estimation method in the presence of either uniform phase noise or phase diffusion are also investigated and discussed.

Keywords: optical implementation of quantum information processing, interference, photon statistics, photon-number resolving detectors

1. INTRODUCTION

Communication channels based on coherent signals are of great interest for the scientific community in view of their technological applications when quantum resources, such as entanglement, cannot be exploited.[1] Actually, because of the non-orthogonal character of coherent states, such channels require the development of optimal discrimination strategies.[2,3] For instance, in the case of a binary phase-shift-keyed (BPSK) channel involving two coherent states, the detection/discrimination stage includes a quasi-optimal receiver, in which a local oscillator (LO) is mixed at a beam splitter with the states to be discriminated. The reliability of such systems is limited by the need of knowing the relative phase between signals and LO.[4] To deal with this issue, here we propose, and experimentally test, a real-time monitoring of the relative phase by considering one of the best known BPSK communication receivers, namely the Kennedy-like receiver.[5–7] In the traditional scheme, the two coherent signals encoding the message interfere with a reference LO and the output is detected with an ON/OFF photodetector. As already shown in [8], here we demonstrate that, without interrupting the communication, the Bayesian processing of a set of data samples, corresponding to a statistical mixture of the states to be discriminated, allows the achievement of the minimum uncertainty in phase estimation, namely the inverse of the Fisher information associated with the statistics of the collected data. Moreover, we show that the use of photon-number resolving detectors in the receiver enhances phase-estimation strategy with respect to the usually employed ON/OFF detectors. The performances of our phase-estimation method are also investigated, both numerically and experimentally, in the presence of a uniform phase noise, whereas considerations about the phenomenon of phase diffusion as a noise source are discussed here for the first time from the theoretical point of view.

Further author information: (Send correspondence to A.A.)
A.A.: E-mail: alessia.allevi@uninsubria.it, Telephone: +39 031 2386253

Quantum Optics and Quantum Information Transfer and Processing 2015,
edited by Konrad Banaszek, Christine Silberhorn, Proc. of SPIE Vol. 9505,
95050J · © 2015 SPIE · CCC code: 0277-786X/15/$18 · doi: 10.1117/12.2178937

2. THEORY

Without loss of generality, we assume that the two coherent signals $|\pm\beta\rangle$ used in the BPSK communication scheme are sent with the same prior probability, $z_0 = z_1 = 1/2$. The overall state reaching the receiver is a phase-sensitive statistical mixture of the two coherent states, which can be described by the following density operator:

$$\hat{\varrho}(\beta) = \frac{1}{2}\left(|\beta\rangle\langle\beta| + |-\beta\rangle\langle-\beta|\right). \tag{1}$$

To achieve shot-by-shot quasi-optimal discrimination[9] with a Kennedy-like receiver, the state $\hat{\varrho}(\beta)$ is then mixed at a beam splitter (BS) of transmittance τ with a LO excited in the coherent state $|\alpha\rangle$.
The overall output state can be written as $\hat{\varrho}_{\text{out}}(a,b,\phi) = \hat{D}(a\,\mathrm{e}^{\mathrm{i}\phi})\hat{\varrho}(b)\hat{D}^{\dagger}(a\,\mathrm{e}^{\mathrm{i}\phi})$, where $a = \alpha\sqrt{1-\tau}$ and $b = \beta\sqrt{\tau}$. The photon-number distribution of the transmitted beam reads

$$p_n(a,b,\phi) = \frac{1}{2}\left(\mathrm{e}^{-\nu_+}\frac{\nu_+^n}{n!} + \mathrm{e}^{-\nu_-}\frac{\nu_-^n}{n!}\right), \tag{2}$$

which is the sum of two Poisson distributions depending on ϕ through the mean values $\nu_\pm = a^2 + b^2 \pm 2\,a\,b\cos\phi$. Therefore, the real-time phase monitoring can be directly obtained from Eq. (2) by using the same acquired data and a suitable estimation strategy. In Ref. [8], we have already demonstrated that, without the need of stopping the communication, by suitably selecting consecutive data samples and calculating their probability distribution $P(\{n_k\}|\phi)$, the Bayes theorem allows us to obtain the posterior probability $P(\phi|\{n_k\})$ of ϕ given the sample. In particular, we have shown that the estimator of the phase ϕ is $\bar{\phi} = \int d\phi\,\phi\,P(\phi|\{n_k\})$, and its variance is $\mathrm{Var}_\phi = \int d\phi\,(\phi-\bar{\phi})^2\,P_{\text{PNR}}(\phi|\{n_k\})$. It is well known[10,11] that the Bayes estimator is asymptotically optimal if $M \gg 1$, that is $\mathrm{Var}_\phi \to [M\,F_\phi]^{-1}$, where $F_\phi = \sum_n p_n(a,b,\phi)[\partial_\phi \ln p_n(a,b,\phi)]^2$ is the Fisher information associated with $p_n(a,b,\phi)$.
In principle, all these results can be achieved by means of ON/OFF detectors, namely detectors that can simply discriminate between the presence and the absence of light. However, in our previous work we have demonstrated that the use of a photon-number resolving (PNR) detector instead of an ON/OFF detector is better as the amount of Fisher information that can be extracted is larger, the estimator $\bar{\phi}$ converges more rapidly and the variance is lower. For the sake of clarity, in Fig. 1 we plot the results of a Monte Carlo simulation, in which

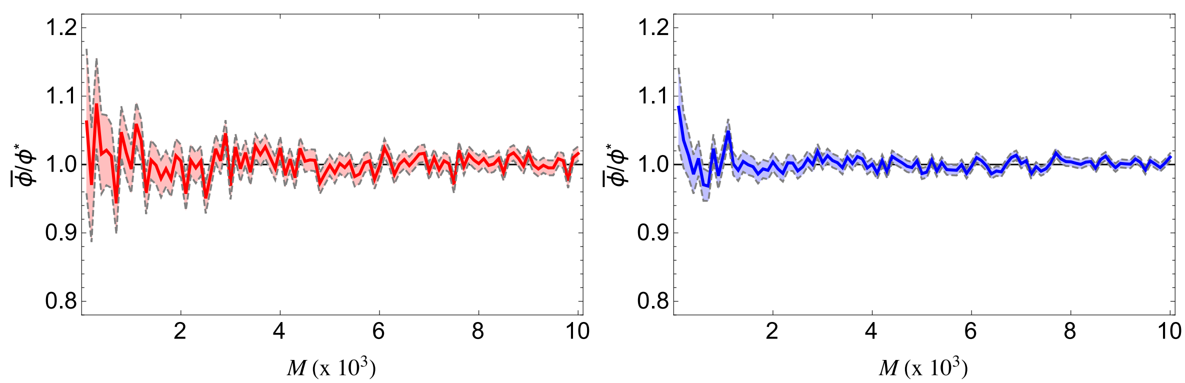

Figure 1. (Color online) Bayesian estimator in numerical simulations by exploiting ON/OFF detectors (left panel) and PNR detectors (right panel). The plots show the ratio $\bar{\phi}/\phi^*$ (solid curve) and the corresponding standard deviation (dashed curve) as functions of the number of data M. We set $a = b = \sqrt{2}$ and $\phi^* = 0.3$.

the same Bayesian strategy was applied either to ON/OFF or PNR detectors. In the left and right panels we present the ratio between the Bayesian estimator $\bar{\phi}$ and the actual value of the phase ϕ^* for the two detection schemes. As it is evident from the figure, the use of PNR detector brings $\bar{\phi}$ converge more rapidly to the value of $\phi^* = 0.3$ (just after $M \sim 10^3$ data) than using the other scheme. The same conclusion can be drawn by looking at Fig. 2, in which the variance of ϕ is shown as a function of the number of data for both detectors.

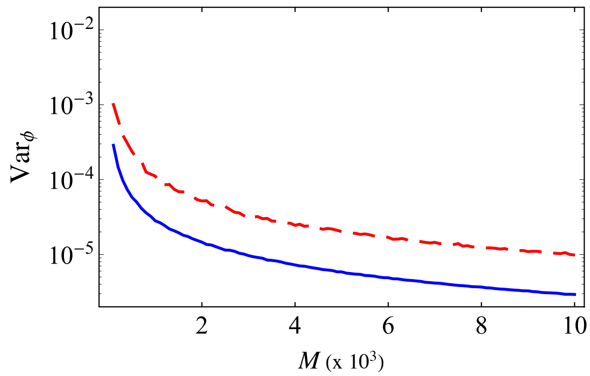

Figure 2. (Color online) Logarithmic plot of the variance Var_ϕ as a function of the number of data M for ON/OFF detectors (red, dotted) and PNR detectors (blue, solid). We set $a = b = \sqrt{2}$ and $\phi^* = 0.3$.

In a more realistic scenario, the input coherent states can be affected by phase noise during generation and propagation.[12,13] In the presence of uniform phase noise, the degraded state is the so-called bracket $\hat{\varrho}(b,\gamma) \equiv \gamma^{-1} \int_{-\gamma/2}^{\gamma/2} d\psi\, \hat{\varrho}(b\,\mathrm{e}^{\mathrm{i}\psi})$,[14] where γ is the noise parameter. Also in this case the Bayesian approach, applied to the photon-number probability distribution $p_n(a,b,\phi,\gamma) \equiv \gamma^{-1} \int_{-\gamma/2}^{\gamma/2} d\psi\, p_n(a,b,\phi-\psi)$ of the output state $\hat{\varrho}_{\mathrm{out}}(a,b,\phi,\gamma) = \hat{D}(a\,\mathrm{e}^{\mathrm{i}\phi})\hat{\varrho}(b,\gamma)\hat{D}^\dagger(a\,\mathrm{e}^{\mathrm{i}\phi})$, results very robust in the phase estimation, only showing small differences in the convergence compared to the ideal case of vanishing noise ($\gamma \to 0$). As expected, the effect of the uniform phase noise described by the bracket states is a slight increase of the variance of the estimation procedure.[8] Another kind of phase noise is given by phase diffusion.[12] The phase-diffusion process that affects the propagation of the signal $\hat{\varrho} = \hat{\varrho}(b)$ in Eq. (1) can be described by means of the master equation:

$$\frac{\mathrm{d}\hat{\varrho}}{\mathrm{d}t} = \frac{\Gamma}{2}\,\mathcal{L}[\hat{a}^\dagger\hat{a}]\hat{\varrho}, \tag{3}$$

where $\mathcal{L}[\hat{O}]\hat{\varrho} = 2\hat{O}\hat{\varrho}\hat{O}^\dagger - \hat{O}^\dagger\hat{O}\hat{\varrho} - \hat{\varrho}\hat{O}^\dagger\hat{O}$ and Γ is the phase-damping rate. If we introduce $\sigma^2 = \Gamma t/2$, the evolved state $\hat{\varrho}^{(\mathrm{pd})}(b,\sigma)$ at time t can be written as:[15]

$$\hat{\varrho}^{(\mathrm{pd})}(b,\sigma) = \int_{\mathbb{R}} \mathrm{d}\varphi\, g(\varphi,\sigma)\,\hat{U}_\varphi\,\hat{\varrho}(b)\,\hat{U}_\varphi^\dagger, \quad g(\varphi,\sigma) = \frac{\mathrm{e}^{-\varphi^2/(2\sigma^2)}}{\sqrt{2\pi\sigma^2}}, \tag{4}$$

where $\hat{U}_\varphi = \exp(-i\varphi\,\hat{a}^\dagger\hat{a})$ is the phase-shift operator. Thus, the state impinging on the detector after the

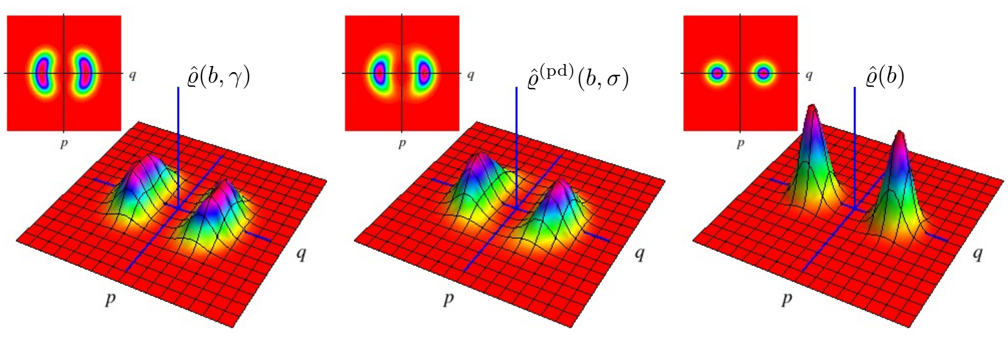

Figure 3. (Color online) Wigner functions of the states $\hat{\varrho}(b,\gamma)$ (left) and $\hat{\varrho}^{(\mathrm{pd})}(b,\sigma)$ (center) affected by uniform noise and phase diffusion, respectively. In the right panel the Wigner function of the corresponding state $\hat{\varrho}(b)$ in the absence of noise is shown. We set $\gamma = \pi/2$, $\sigma^2 = \gamma^2/12$, and $b = 2.0$.

interference with the LO can be written as: $\hat{\varrho}^{(\mathrm{pd})}_{\mathrm{out}}(a, b, \phi; \sigma) = \hat{D}(a\,\mathrm{e}^{\mathrm{i}\phi})\hat{\varrho}(b, \sigma)\hat{D}^{\dagger}(a\,\mathrm{e}^{\mathrm{i}\phi})$. In order to compare the effect of uniform noise, presented above, with the phase diffusion process, we set $\sigma^2 = \gamma^2/12$. Indeed, with such an assumption the Gaussian distribution $g(\varphi, \sigma)$ characterizing the phase-diffusion process and the uniform distribution within the interval $[-\gamma/2, \gamma/2]$ have the same first two moments. For the sake of clarity, in Fig. 3 we plot the Wigner functions of the states $\hat{\varrho}(b, \gamma)$ for $\gamma = \pi/2$, $\hat{\varrho}^{(\mathrm{pd})}(b, \sigma)$ for $\sigma^2 = \gamma^2/12$, and $\hat{\varrho}(b)$, *i.e.* the mixture of coherent states in the absence of noise. Moreover, in Fig. 4 we compare the two types of noise by varying the noise parameter γ (and consequently σ). As the noise parameter increases, the detrimental effect of uniform noise (especially for ON/OFF detectors) with respect to Gaussian phase diffusion becomes more evident.

Finally, in Fig. 5 we plot the variance Var_ϕ as a function of the number of data M for two different choices

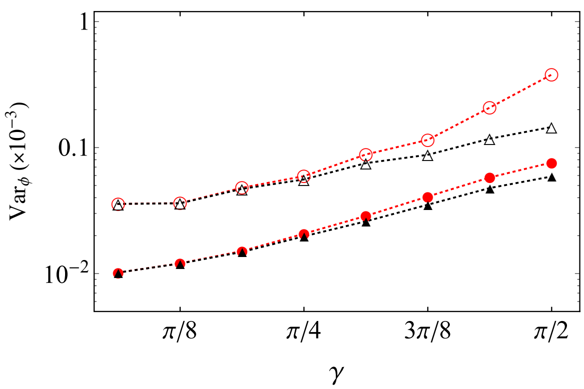

Figure 4. (Color online) Logarithmic plot of Var_ϕ in the presence of uniform noise (red disks) and phase diffusion (black triangles) as a function of γ (we recall that $\sigma^2 = \gamma^2/12$), at fixed number of data $M = 3 \times 10^3$. Full and open symbols refer to PNR and ON/OFF detection, respectively. Simulated setup parameters: $\phi^* = 0.3$, $a = b = \sqrt{2}$.

of the noise parameter γ, both for ON/OFF and PNR detectors. We can conclude that the estimation method works well also in the presence of a phase-diffusion process. Nevertheless, the better performances (*i.e.* lower variance values) in the case of phase diffusion instead of uniform phase noise lead us to conclude that the most detrimental situation to test the validity of our phase-estimation strategy is the latter.

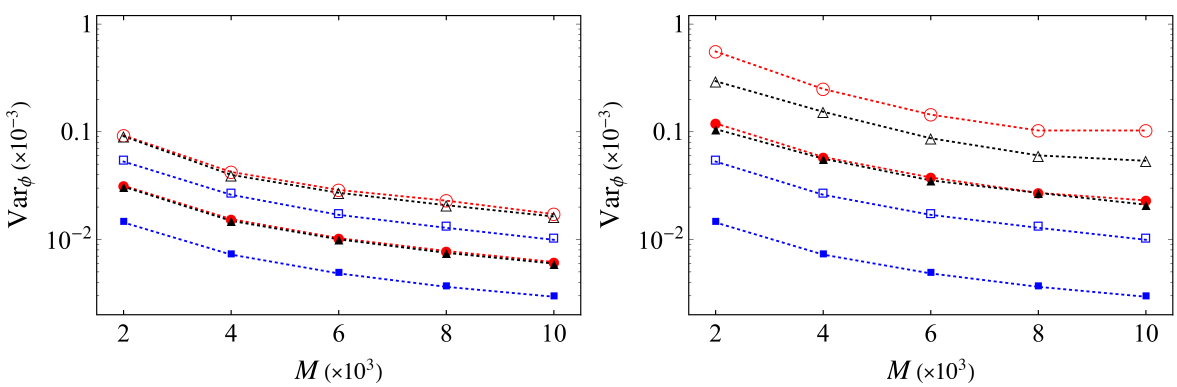

Figure 5. (Color online) Logarithmic plot of the variance Var_ϕ as a function of the number of data M for input states $\hat{\varrho}(b, \gamma)$ (red disks), $\hat{\varrho}^{(\mathrm{pd})}(b, \sigma)$ (black triangles), and $\hat{\varrho}(b)$ (blue squares). Full and open symbols refer to PNR and ON/OFF detection, respectively. Simulated setup parameters: $\phi^* = 0.3$, $a = b = \sqrt{2}$, $\gamma = \pi/4$ (left panel) and $\gamma = \pi/2$ (right panel). We set $\sigma^2 = \gamma^2/12$.

3. EXPERIMENTAL RESULTS

To experimentally validate our analysis, we realized a proof-of-principle scheme. The bracket states were gen-

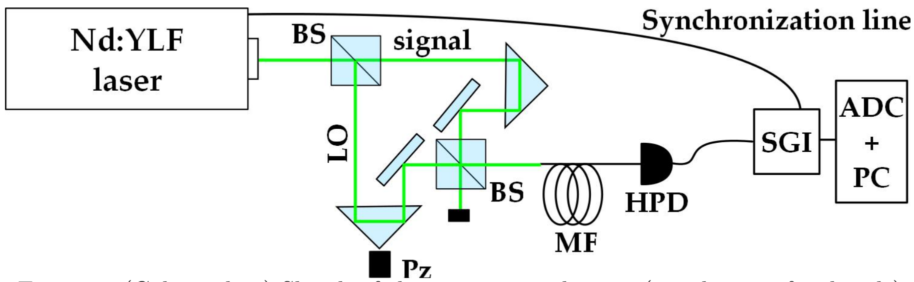

Figure 6. (Color online) Sketch of the experimental setup (see the text for details).

erated by exploiting the second-harmonic pulses (5-ps pulse duration) at 523 nm of a mode-locked Nd:YLF laser regeneratively amplified at 500 Hz (High-Q Laser Production). The pulses were sent to a Mach-Zehnder interferometer, in which the relative phase between the two arms (signal and LO in Fig. 6) was changed in steps by means of a piezoelectric movement (Pz).[16] The light exiting the interferometer was delivered through a multi-mode fiber (MF, 600-μm-core diameter) to a PNR detector, namely a hybrid photodetector (HPD, R10467U-40, maximum quantum efficiency ~ 0.5 at 500 nm, Hamamatsu). The output of the detector was then amplified, synchronously integrated (SGI) and digitized (ADC). As already explained in Refs. [16-18], with our detection system we are able not only to reconstruct the statistics of detected photons, but also to determine the actual

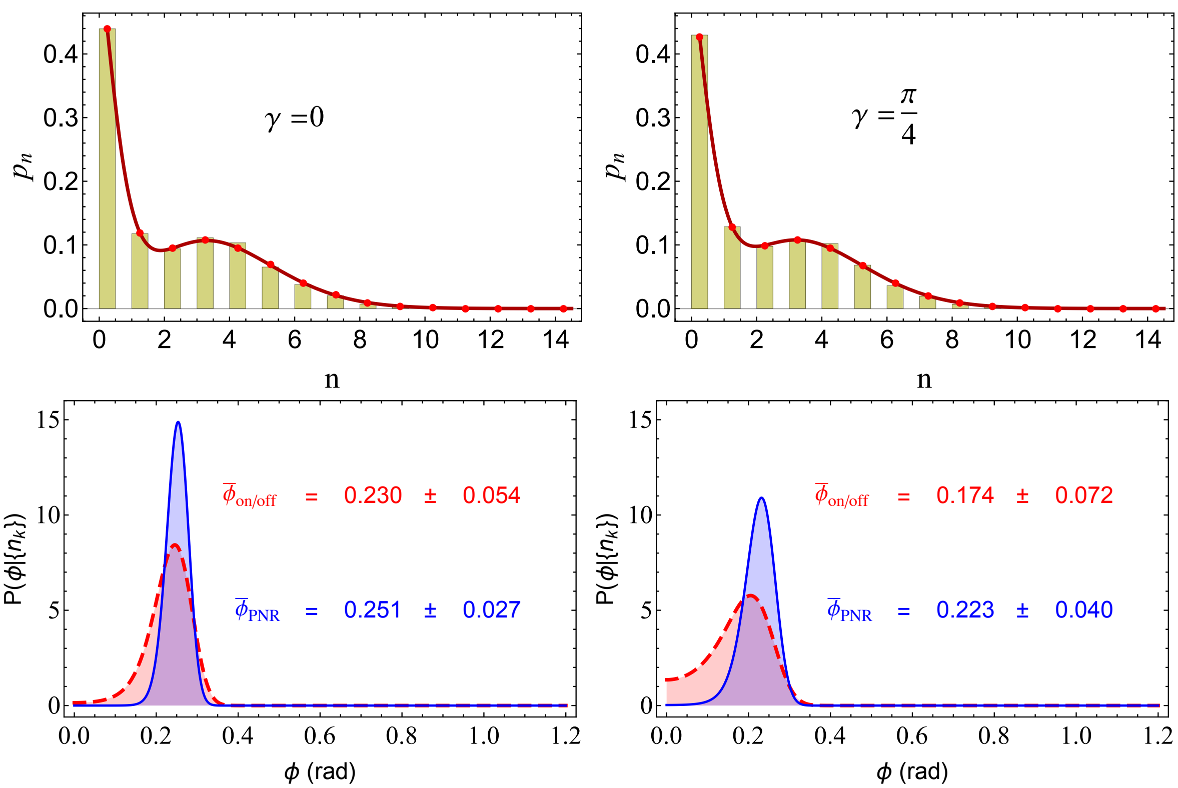

Figure 7. (Color online) Top: Experimental $p_n^{(\rm exp)}$ distributions obtained by means of PNR detection (dark yellow histograms) and theoretical expectations $p_n\left(a, b, \bar{\phi}_{\rm PNR}, \gamma\right)$ (red lines). Bottom: Posterior probabilities of ϕ for $M = 4\,000$ with ON/OFF (dashed red curve) and PNR (solid blue curve) detectors. The experimental parameters are $a = 1.12$, $b = 0.79$, $\phi^* \simeq 0.25$, $\gamma = 0$ (left panels) and $\gamma = \pi/4$ (right panels).

value of the phase ϕ at each piezo position, independent of the regularity and reproducibility of the movement. In fact, by monitoring the mean number of detected photons as a function of the piezoelectric movement, an interference pattern emerges, from which the value of ϕ can be estimated.[14,16] Once the relative phase is determined, the simulation of bracket states with a uniform phase noise can be achieved by combining a set of data characterized by a central phase ϕ and a uniform integration interval γ and appending it to a second set corresponding to an interval with the same amplitude but with opposite phase. In order to approach the operation of a real communication channel, a numerical randomization of data in the bracket states, followed by an estimation algorithm, is performed. We notice that, in principle, the same result can be experimentally achieved by randomizing the piezoelectric movement, even if spurious effects such as the hysteresis of the device can limit the performances of the system. However, it is worth noting that the randomization is not fundamental in our protocol, since the Bayesian probability depends on the data sample, but not on the actual order of the data. In the following we consider the states corresponding to $\phi^* = 0.25$, with $\gamma = \pi/4$ and $\gamma = \pi/2$, respectively. The results are presented in Fig. 7: In the upper panels we plot the measured photon-number distributions p_n together with the theoretical distributions $p_n(a, b, \bar{\phi}_{\mathrm{PNR}}, \gamma)$, which have fidelities $F = \int_n \sqrt{p_n(a, b, \bar{\phi}_{\mathrm{PNR}}, \gamma)\, p_n^{(\mathrm{exp})}}$ higher than 99.9% to the experimental data.[19] In the lower panels of Fig. 7 we show the posterior probabilities $P_{\mathrm{ON/OFF}}(\phi|n_k)$ and $P_{\mathrm{PNR}}(\phi|n_k)$ with the corresponding estimated phases. In both panels it is evident that, for such a choice of ϕ^*, the posterior probabilities are affected by a bias, testified by the asymmetric shape of the curve. Indeed, this situation enhances the better estimation achieved by means of PNR detection instead of ON/OFF scheme, as the posterior probability is more peaked, has smaller variance and reduced asymmetry.

4. CONCLUSIONS

We proposed and realized a real-time method to monitor the phase reference of a Kennedy-like receiver without stopping the communication. The theoretical model was not only proved by numerical simulations, but also validated by a proof-of principle experiment. Our strategy, based on Bayesian analysis, can asymptotically reach the minimum uncertainty in the estimation just after few thousands of data, also in the presence of noise. We compared the effectiveness of our estimation method in the case of uniform phase noise as well as in the presence of phase diffusion. Furthermore, the advantage of using PNR detectors with respect to ON/OFF detectors has been theoretically predicted and also verified by means of simulations and experimental results.

From the one hand our investigation opens new perspectives in the field of Quantum Communication as it represents a first step toward making the Kennedy-like receiver a practical useful technology. From the other hand, our strategy can find applications also in other contexts, in which the estimation of the phase represents an unavoidable requirement.

ACKNOWLEDGMENTS

Support by MIUR (FIRB "LiCHIS" - RBFR10YQ3H) is acknowledged.

REFERENCES

[1] Agarwal, G. S., *Fiber-Optic Communication Systems* (Wiley, 2010).

[2] Becerra, F. E., Fan, J., Baumgartner, G., Goldhar, J., Kosloski, J. T. and Migdall, A., "Experimental demonstration of a receiver beating the standard quantum limit for multiple nonorthogonal state discrimination," Nature Photon. 7, 147 (2013).

[3] Chefles, A., "Quantum state discrimination," Contemp. Phys. 41(6), 401-424 (2000).

[4] Muller, C. R., Wittmann, C., Marek, P., Filip, R., Marquardt, Ch., Leuchs, G. and Andersen, U. L., "Probabilistic cloning of coherent states without a phase reference," Phys. Rev. A 86(1), 010305(R) (2012).

[5] Kennedy, R. S., Hoversten, E. V., Elias, P. and Chan, V., "Processing and transmission of information," MIT RLE Quarterly Progress Report, *N. 108* (1973) unpublished.

[6] Wittmann, C., Takeoka, M., Cassemiro, K. N., Sasaki, M., Leuchs, G. and Andersen, U. L., "Demonstration of near-optimal discrimination of optical coherent states," Phys. Rev. Lett. 101(21), 210501 (2008).

[7] Tsujino, K., Fukuda, D., Fujii, G., Inoue, S., Fujiwara, M., Takeoka, M. and Sasaki, M., "Quantum receiver beyond the standard quantum limit of coherent optical communication," Phys. Rev. Lett. 106(25), 250503 (2011).

[8] Bina, M., Allevi, A., Bondani, M. and Olivares, S.,"Real-time phase-reference monitoring in quasi-optimal coherent-state discrimination," submitted and quant-ph/1408.0228.

[9] Olivares, S. and Paris, M. G. A., "Binary optical communication in single-mode and entangled quantum noisy channels," J. Opt. B: Quantum Semiclass. Opt. 6(1), 69 (2004).

[10] Teklu, B., Olivares, S. and Paris, M. G. A., "Bayesian estimation of one-parameter qubit gates," J. Phys. B: At. Mol. Opt. Phys. 42(3), 035502 (2009).

[11] Olivares, S. and Paris, M. G. A., "Bayesian estimation in homodyne interferometry," J. Phys. B: At. Mol. Opt. Phys. 42(5), 055506 (2009).

[12] Genoni, M. G., Olivares, S. and Paris, M. G. A., "Optical phase estimation in the presence of phase diffusion," Phys. Rev. Lett. 106(15), 153603 (2011).

[13] Olivares, S., Cialdi, S., Castelli, F. and Paris, M. G. A., "Homodyne detection as a near-optimum receiver for phase-shift-keyed binary communication in the presence of phase diffusion," Phys. Rev. A 87(5), 050303(R) (2013).

[14] Allevi, A., Olivares, S. and Bondani, M., "Bracket states for communication protocols with coherent states," Int. J. Quant. Inf. 12(2), 1461018 (2014).

[15] Genoni, M. G., Olivares, S., Brivio, D., Cialdi, S., Cipriani, D., Santamato, A., Vezzoli, S. and Paris, M. G. A., "Optical interferometry in the presence of large phase diffusion," Phys. Rev. A 85(4), 043817 (2012).

[16] Bondani, M., Allevi, A. and Andreoni, A., "Self-consistent phase determination for Wigner function reconstruction," J. Opt. Soc. Am. B 27(2), 333-337 (2010).

[17] Bondani, M., Allevi, A., Agliati, A. and Andreoni, A., "Self-consistent characterization of light statistics," J. Mod. Opt. 56(2-3), 226-231 (2009).

[18] Bondani, M., Allevi, A. and Andreoni, A., "Light statistics by non-calibrated linear photodetectors," Adv. Sci. Lett. 2(4), 463-468 (2009).

[19] Bina, M., Mandarino, A., Olivares, S. and Paris, M. G. A., "Drawbacks of the use of fidelity to assess quantum resources," Phys. Rev. A 89(1), 012305 (2014).